

Matter To The Deepest 2015

(* Time of evaluation and the version of the *Mathematica* kernel used *)

DateString[]

Wed 23 Sep 2015 14:52:33

\$Version

9.0 for Linux x86 (64-bit) (November 20, 2012)

<< **MB.m**

MB 1.2

by Michal Czakon

improvements by Alexander Smirnov

more info in hep-ph/0511200

last modified 2 Jan 09

<< **MBsums.v1.0.m**

MBsums v1.0 by Michal Ochman

The author would like to thank Tord Riemann

for many fruitful discussions

```
int = MBint[-((-x)z1+z6 yz2 Gamma[-z1] Gamma[1+z1] Gamma[-z1-z2] Gamma[-z2] Gamma[z2]
Gamma[-z1+z2] Gamma[-z2-z6] Gamma[z2-z6] Gamma[-z6] Gamma[1+z6]) /
(2 eps Gamma[-2 z1] Gamma[1-z2] Gamma[1+z2] Gamma[-2 z6]),
{{eps -> 0}, {z1 -> -1/2, z2 -> -11/192, z6 -> -1/2}}]
```

```
MBint[-((-x)z1+z6 yz2 Gamma[-z1] Gamma[1+z1] Gamma[-z1-z2] Gamma[-z2] Gamma[z2]
Gamma[-z1+z2] Gamma[-z2-z6] Gamma[z2-z6] Gamma[-z6] Gamma[1+z6]) /
(2 eps Gamma[-2 z1] Gamma[1-z2] Gamma[1+z2] Gamma[-2 z6]),
{{eps -> 0}, {z1 -> -1/2, z2 -> -11/192, z6 -> -1/2}}]
```

```
Lk = {x -> -5, y -> 7};
```

```
sums = MBIntToSum[int, {x -> -5, y -> 7}, {z1 -> L, z6 -> L, z2 -> L}]
```

```

z1->L ( Re z1 < -1/2 )
z6->L ( Re z6 < -1/2 )
z2->L ( Re z2 < -11/192 )
{MBSum[
  - ((-1)^-2 n3 (-x)^-n1-n2 y^-n3 ((-1+n3)!)^2 (n1+n3)! (n2+n3)! (2 HarmonicNumber[-1+n3] -
    2 HarmonicNumber[n3] - HarmonicNumber[-1-n1+n3] + HarmonicNumber[n1+n3] -
    HarmonicNumber[-1-n2+n3] + HarmonicNumber[n2+n3] - Log[y])) /
  (2 eps x^2 (1+2 n1)! (1+2 n2)! (n3!)^2 (-1-n1+n3)! (-1-n2+n3)!),
  n1 ≥ 0 && n2 ≥ 0 && n3 ≥ 1 && 1+n1 ≤ n3 && 1+n2 ≤ n3,
  {n1, n2, n3}],
MBSum[- ((-1)^-n1-n3 (-x)^-n1-n2 y^-n3 (n1-n3)! ((-1+n3)!)^2 (n1+n3)! (n2+n3)!) /
  (2 eps x^2 (1+2 n1)! (1+2 n2)! (n3!)^2 (-1-n2+n3)!),
  n1 > 0 && n3 ≥ 1 && n2 ≥ 0 && n1 ≥ n3 && 1+n2 ≤ n3, {n1, n2, n3}],
MBSum[- ((-1)^-n2-n3 (-x)^-n1-n2 y^-n3 (n2-n3)! ((-1+n3)!)^2 (n1+n3)! (n2+n3)!) /
  (2 eps x^2 (1+2 n1)! (1+2 n2)! (n3!)^2 (-1-n1+n3)!),
  n1 ≥ 0 && n2 > 0 && n3 ≥ 1 && n2 ≥ n3 && 1+n1 ≤ n3, {n1, n2, n3}]]}

```

MBintegrate[{int}, Lk]

Shifting contours...

Performing 0 lower-dimensional integrations with NIntegrate

Higher-dimensional integrals

Preparing MBpartleps-1 (dim 3)

Running MBpartleps-1

$$\left\{ \frac{0.00621896}{\text{eps}}, \left\{ \frac{6.21756 \times 10^{-7}}{\text{eps}}, 0 \right\} \right\}$$

DoAllMBSums[sums, 25, Lk] // N

$$\frac{0.00621915}{\text{eps}}$$

ToString[int, InputForm, PageWidth → 50]

```

MBint[-((-x)^(z1+z6)*y^z2*Gamma[-z1]*
  Gamma[1+z1]*Gamma[-z1-z2]*Gamma[-z2]*
  Gamma[z2]*Gamma[-z1+z2]*Gamma[-z2-z6]*
  Gamma[z2-z6]*Gamma[-z6]*Gamma[1+z6])/
  (2*eps*Gamma[-2*z1]*Gamma[1-z2]*Gamma[1+z2]*
  Gamma[-2*z6]), {{eps -> 0},
  {z1 -> -1/2, z2 -> -11/192, z6 -> -1/2}}]

```

Tostring[sums, InputForm, PageWidth → 50]

```
{MBSum[-((-x)^(-n1 - n2)*(-1 + n3)!^2*(n1 + n3)!*
  (n2 + n3)!*(2*HarmonicNumber[-1 + n3] -
  2*HarmonicNumber[n3] - HarmonicNumber[
  -1 - n1 + n3] + HarmonicNumber[n1 + n3] -
  HarmonicNumber[-1 - n2 + n3] +
  HarmonicNumber[n2 + n3] - Log[y]))/
  (2*(-1)^(2*n3)*eps*x^2*y^n3*(1 + 2*n1)!*
  (1 + 2*n2)!*n3!^2*(-1 - n1 + n3)!*
  (-1 - n2 + n3)!), n1 >= 0 && n2 >= 0 &&
  n3 >= 1 && 1 + n1 <= n3 && 1 + n2 <= n3,
  {n1, n2, n3}],
MBSum[-((-1)^(-n1 - n3)*(-x)^(-n1 - n2)*
  (n1 - n3)!*(-1 + n3)!^2*(n1 + n3)!*
  (n2 + n3)!)/(2*eps*x^2*y^n3*(1 + 2*n1)!*
  (1 + 2*n2)!*n3!^2*(-1 - n2 + n3)!),
  n1 > 0 && n3 >= 1 && n2 >= 0 && n1 >= n3 &&
  1 + n2 <= n3, {n1, n2, n3}],
MBSum[-((-1)^(-n2 - n3)*(-x)^(-n1 - n2)*
  (n2 - n3)!*(-1 + n3)!^2*(n1 + n3)!*
  (n2 + n3)!)/(2*eps*x^2*y^n3*(1 + 2*n1)!*
  (1 + 2*n2)!*n3!^2*(-1 - n1 + n3)!),
  n1 >= 0 && n2 > 0 && n3 >= 1 && n2 >= n3 &&
  1 + n1 <= n3, {n1, n2, n3}]}
```