Two-loop Bhabha scattering with $N_f = 2$

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Loops and Legs, 23th April 2008, Sondershausen

Outline

The Luminosity Monitor

Luminosity of a collider depends on the machine and the beam: all is complicated (not mentioning errors estimation)

$$\frac{dN}{dt} = \mathcal{L}(t), \quad N = \sigma \int dt \mathcal{L}(t) = \sigma \mathcal{L} \tag{1}$$

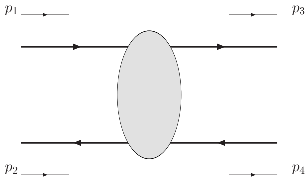
better is to choose a well known process:

$$\mathcal{L} = \frac{1}{\sigma_{Bhabha}} N_{Bhabha}$$

then we may determine any cross section in (1)

Kinematical Regions for Bhabha

Two regions where the Bhabha-scattering cross section is large and QED dominated



$$s = (p_1+p_2)^2 = 4E^2 > 4m_e^2, \quad t = (p_1-p_3)^2 = -4\,(E^2-m_e^2)\sin^2\frac{\theta}{2} < 0$$

- $\sqrt{s} \sim 10^2$ GeV $\Rightarrow$ small θ
- SABS $\Rightarrow \mathcal{L}$ at LEP, ILC

  $\sim$ a few degrees

- $\sqrt{s} \sim$ 1-10 GeV $\Rightarrow$ large θ
- LABS $\Rightarrow \mathcal{L}$ at KLOE

  $\theta \sim 55^0 - 125^0$

Experimental Precision on $\mathcal{L}$

LABS $\rightarrow 10^{-3}$

- KLOE $\Rightarrow \frac{\delta\mathcal{L}}{\mathcal{L}}|_{\rm exp} = 0.3\%$
  DAΦNE-VEPP-2M $\Rightarrow$ 0.1 %

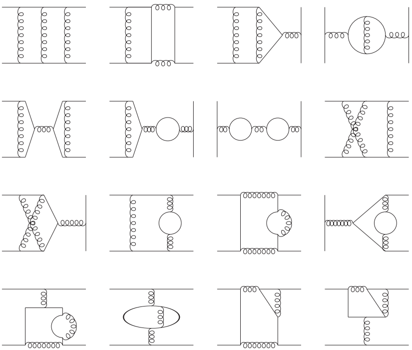
SABS $\rightarrow 10^{-4}$

- LEP $\Rightarrow \frac{\delta\mathcal{L}}{\mathcal{L}}|_{\rm exp} = 0.03\%$
  ILC $\Rightarrow \frac{\delta\mathcal{L}}{\mathcal{L}}|_{\rm exp} = 0.02(1)\%$ Giga-Z

Status around 2004, from S. Jadach, hep-ph:0306083

Type of correction/error	LEPEWWG	hep-ph:9905235	update
Technical precision	–	– (0.03%)	**0.03%**
Missing photonic $\mathcal{O}(\alpha^2 L)$	0.10%	0.027% (0.013%)	**0.013%**
Missing photonic $\mathcal{O}(\alpha^3 L^3)$	0.015%	0.015% (0.006%)	**0.006%**
Vacuum polarization	0.04%	0.040%	**0.025%**
Light pairs	0.03%	0.010%	**0.010%**
Z-exchange	0.015%	0.015%	**0.015%**
Total	0.11%	0.054% (0.055%)	**0.045%**

NNLO QED Corrections [Bern, Dixon, Ghinculov '00]



from Bern,Dixon,Ghinculov, `arXiv:hep-ph/0010075`

For practical applications

$$\sigma = \int \mathrm{d}\Phi\, 2\, \mathrm{Re}\left[\mathcal{M}^{\mathrm{2loop}} \cdot \mathcal{M}^{\mathrm{tree}*}\right]$$

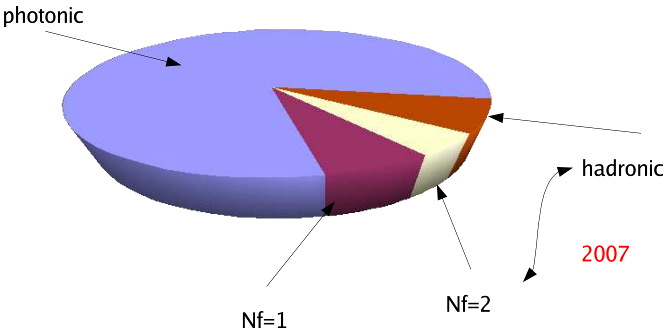
Simplification $m_e = 0$

- + 2-scale problem s, t
- + simple result $\ln(x)$, $\mathrm{Li}_n(x)$
  $x = -t/s = \sin^2(\theta/2)$
- - bad for MCs, $m_e \neq 0$

$\Rightarrow$ Recompute with $m_e \neq 0$

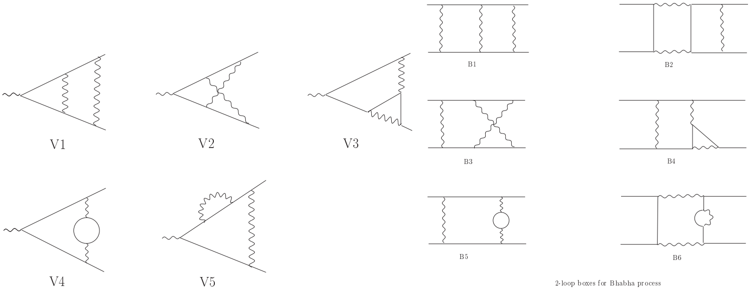
$\Rightarrow$ Massive from massless

Progress since last Loops and Legs, 2006



Remarkable: photonic, $N_f = 1, N_f = 2$ NNLO corrections doubly (triply) cross-checked

NNLO photonic and fermionic $N_f = 1, 2$ topologies



- SE loop insertions (without photonic line) are so called fermionic diagrams, rest represents photonic.
- Closed fermionic loop can be muon, tau, top or light quarks.
- In general, box B5 is a 4-scale problem: $m_e, m_f, s, t(u)$.

Photonic NNLO massive QED Corrections

Photonic corrections (no fermion loop)

$$\frac{d\sigma_2}{d\sigma_0} = \underbrace{A\ \ln^2\left(\frac{s}{m_e^2}\right) + B\ \ln\left(\frac{s}{m_e^2}\right)}_{\text{collinear}} + \underbrace{C}_{\text{non collinear}}$$

- [Arbuzov,Kuraev,Shaikhatdenov '98] [Glover,Tausk,van der Bij '01] $\Rightarrow$ A, B
- [Penin '05] $\Rightarrow$ C=C$_0$ + $\mathcal{O}(m_e^2/s)$ from massless result

- Result of A. Penin confirmed by independent calculation

[Becher-Melnikov '07]

Heavy Fermions with IR Matching excluding $\mathcal{O}(m_e^2/s)$

[Penin '05]: photonic corrections follow from a change in the regularization scheme

$$\mathcal{M}(m=0,\ \underbrace{1/\epsilon}_{\text{IR and collinear}}\ ) \Rightarrow \mathcal{M}(\underbrace{\ln(s/m_e^2)}_{\text{collinear}}, \underbrace{\ln(\lambda^2/m_e^2)}_{\text{IR}}) + \delta\mathcal{M}$$

[Mitov-Moch '06]: for $2 \to n$ QED/QCD scattering process

$$\mathcal{M}(m \neq 0) = \prod_j \mathcal{Z}_j^{1/2} \mathcal{M}(m=0) \quad \mathcal{Z}_j = \frac{F_D(Q^2, m \neq 0)}{F_D(Q^2, m=0)}$$

[Becher-Melnikov '07]: in QED assuming $m_e^2 << m_f^2 << s, t, u$ heavy fermions can be included

$$\mathcal{M}(m \neq 0) = \prod_j \mathcal{Z}_j^{1/2} \underbrace{\mathcal{S}(m_f)}_{\text{process dependent}} \mathcal{M}(m=0)$$

$N_f = 1$ NNLO massive QED Corrections

Fermionic corrections: electrons in SE loops

- [Bonciani,Ferroglia,Mastrolia,Remiddi,van der Bij '05]

  $\Rightarrow$ electron loops with full m_e dependence

recalculated in 2007

- [Actis, JG, Czakon, Riemann '07]

  $\Rightarrow$ electron loops with full m_e dependence

and in small electron limit:

- [Becher-Melnikov '07]

  $\Rightarrow$ electron loops $m_e^2 << s, t$

Evaluation of the MIs Electron-Loop Case

MIs evaluated using the method of differential equations

[Kotikov '91] [Remiddi '97] [Caffo,Czyz,Laporta,Remiddi '98]

Three-scale problem: s, t, m_e

$$x = \frac{\sqrt{s} - \sqrt{s - 4m_e^2}}{\sqrt{s} + \sqrt{s - 4m_e^2}} \qquad y = \frac{\sqrt{4m_e^2 - t} - \sqrt{-t}}{\sqrt{4m_e^2 - t} + \sqrt{-t}}$$

MIs: HPLs [Remiddi,Vermaseren '99] and 2dHPLs [Gehrmann,Remiddi '00]

Full agreement with the existing result

[Bonciani,Ferroglia,Mastrolia,Remiddi,van der Bij '05]

[S.A.,Czakon,Gluza,Riemann '07]

$N_f = 2$ NNLO leptonic massive QED Corrections

First results:

- [Becher-Melnikov '07]

  $\Rightarrow m_e^2 << m_f^2 << s, t, u$
- [Actis, JG, Czakon, Riemann '07]

  $\Rightarrow m_e^2 << m_f^2 << s, t, u$

Next talk: “Two-Loop Heavy-Flavor Contribution to Bhabha Scattering”

- [Bonciani, Ferroglia, Penin '07]

  $\Rightarrow$ here: $m_e^2 << m_f^2, s, t, u$

Finally, yet another approach (dispersion relations)

- [Actis, JG, Czakon, Riemann '08]

  $\Rightarrow m_e^2 << m_f^2, s, t, u$

I: Evaluation of the MIs Heavy Fermions

Four-scale problem: s, t, m_e, m_f $\rightarrow$ new heavy-fermion scale

- Exploit the hierarchy of scales $m_e^2 << m_f^2 << s, t, u$
- Evaluate the MIs neglecting $\mathcal{O}(m_e^2/m_f^2)$, $\mathcal{O}(m_e^2/s)$, $\mathcal{O}(m_f^2/s)$

Mellin-Barnes method [Smirnov '99] [Tausk '99] efficient

- $\mathcal{I}^{2L} \sim \underbrace{\int dz \left(\frac{m_e^2}{m_f^2}\right)^{z+\epsilon} \frac{\prod \Gamma_i(z,\epsilon)}{\prod \Gamma_j(z,\epsilon)}}_{\text{integral}} \Rightarrow \underbrace{\sum_k c_k \left(\frac{m_e^2}{m_f^2}\right)^k}_{\text{sum}}$
- MB representations by AMBRE [Kajda, JG, Riemann '07]
- analytical continuation in ϵ with `MB` [Czakon '05]
- sums with `XSummer` [Moch,Uwer '05]

Example: box MIs, $L_m(x) = \ln(-m^2/x)$ and $R = m^2/M^2$

$$
\begin{aligned}
\texttt{B[x,y]} \; = \; & \frac{m^{-4\epsilon}}{x}\Big\{\frac{1}{\epsilon^2}L_m(x) + \frac{1}{\epsilon}\Big(-\zeta_2 + 2L_m(x) + \frac{1}{2}L_m^2(x) + L_m(x)L_m(y)\Big) \\
- \; & 2\zeta_2 - 2\zeta_3 + 4L_m(x) + L_m^2(x) + \frac{1}{3}L_m^3(x) - 4\zeta_2 L_m(y) \\
+ \; & 2L_m(x)L_m(y) + L_m(x)L_m^2(y) - \frac{1}{6}L_m^3(y) \\
- \; & \Big(3\zeta_2 + \frac{1}{2}L_m^2(x) - L_m(x)L_m(y) + \frac{1}{2}L_m^2(y)\Big)\ln\Big(1+\frac{y}{x}\Big) \\
- \; & \Big(L_m(x) - L_m(y)\Big)\mathrm{Li}_2\Big(-\frac{y}{x}\Big) + \mathrm{Li}_3\Big(-\frac{y}{x}\Big)\Big\},
\end{aligned}
$$

$$
\begin{aligned}
\texttt{Bd[x,y]} \; = \; & \frac{m^{-4\epsilon}}{xy}\Big\{\frac{1}{\epsilon}\Big[-L_m(x)L_m(y) + L_m(x)L(R)\Big] - 2\zeta_3 + \zeta_2 L_m(x) + 4\zeta_2 L_m(y) \\
- \; & 2L_m(x)L_m^2(y) + \frac{1}{6}L_m^3(y) - 2\zeta_2 L(R) + 2L_m(x)L_m(y)L(R) - \frac{1}{6}L^3(R) \\
+ \; & \Big(3\zeta_2 + \frac{1}{2}L_m^2(x) - L_m(x)L_m(y) + \frac{1}{2}L_m^2(y)\Big)\ln\Big(1+\frac{y}{x}\Big) \\
+ \; & \Big(L_m(x) - L_m(y)\Big)\mathrm{Li}_2\Big(-\frac{y}{x}\Big) - \mathrm{Li}_3\Big(-\frac{y}{x}\Big)\Big\}.
\end{aligned}
$$

Checks on the Computation

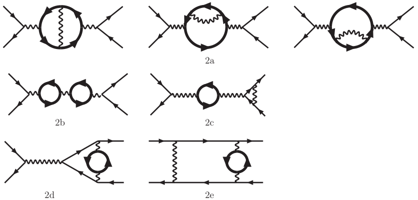
$\Rightarrow$ UV/IR/collinear structure of the result

- UV divergencies cancel after inserting counterterms
- IR divergencies cancel after including single-photon emission
- $\mathcal{O}(m_e^{-n})$ from IBPIs cancel $\Rightarrow$ collinear divergencies $\ln\left(s/m_e^2\right)$

$\Rightarrow$ Dirac FF agrees with [Burgers '85] [Kniehl,Krawczyk,Kühn,Stuart '88]

$\Rightarrow$ Expansions of MIs checked with sector decomposition [Binoth,Heinrich '00]

Structure of the result



SE

reducable diagrams

irreducable diagrams

After combining the 2-loop terms with the loop-by-loop terms and with soft real corrections:

$$\frac{d\sigma^{\mathrm{NNLO}}}{d\Omega} + \frac{d\sigma_{\gamma}^{\mathrm{NLO}}}{d\Omega} = \frac{d\sigma^{\mathrm{NNLO,e}}}{d\Omega} + \sum_{f\neq e} Q_f^2 \frac{d\sigma^{\mathrm{NNLO,f^2}}}{d\Omega} + \sum_{f\neq e} Q_f^4 \frac{d\sigma^{\mathrm{NNLO,f^4}}}{d\Omega} + \sum_{f_1,f_2\neq e} Q_{f_1}^2 Q_{f_2}^2 \frac{d\sigma^{\mathrm{NNLO,2f}}}{d\Omega}.$$

Irreducable terms

$$\frac{d\sigma^{\mathrm{NNLO,f^2}}}{d\Omega} = \frac{\alpha^2}{s}\left\{\sigma_1^{\mathrm{NNLO,f^2}} + \sigma_2^{\mathrm{NNLO,f^2}} \ln\left(\frac{2\omega}{\sqrt{s}}\right)\right\}$$

$$\begin{aligned}\sigma_1^{\mathrm{NNLO,f^2}} &= \frac{\left(1-x+x^2\right)^2}{3x^2}\left\{-\frac{1}{3}\left[\ln^3\left(\frac{s}{m_e^2}\right)+\ln^3\left(\frac{m_e^2}{m_f^2}\right)\right]\right.\\ &+ \ln^2\left(\frac{s}{m_e^2}\right)\left[\frac{55}{6}-\ln\left(\frac{m_e^2}{m_f^2}\right)+\ln\left(-1+1/x\right)\right]\\ &\quad + \cdots\cdots\cdots\end{aligned}$$

Structure of the Result

$$\frac{d\sigma_2}{d\sigma_0} = \left(\frac{\alpha}{\pi}\right)^2 \left[A \ln\left(\frac{2\omega}{\sqrt{s}}\right) + B \ln\left(\frac{s}{m_e^2}\right) + C\right]$$

$$C = C_3 \ln^3\left(\frac{s}{m_f^2}\right) + C_2 \ln^2\left(\frac{s}{m_f^2}\right) + C_1 \ln\left(\frac{s}{m_f^2}\right) + C_0$$

$$C_3 = -\frac{1}{9} \qquad C_2 = \frac{19}{18} + \frac{1}{3}\ln\left(\frac{1-x}{x}\right) \qquad x = -\frac{t}{s} = \sin^2\left(\frac{\theta}{2}\right)$$

$$C_0 \Bigg\} \Rightarrow$$

$$\begin{aligned}
& - \frac{1}{324(1-x+x^2)^2}\Big\{-4795 + 9590x - 14385x^2 + 9590x^3 - 4795x^4 - 1404\zeta_2 + 1674x\,\zeta_2 \\
& + 2484x^2\zeta_2 - 4158x^3\zeta_2 + 2808x^4\zeta_2 + 432\,\zeta_3 - 864x\zeta_3 + 1296x^2\zeta_3 - 864x^3\zeta_3 + 432x^4\zeta_3 \\
& - 672\,L_{-x} + 966xL_{-x} - 2016x^2L_{-x} + 966x^3\,L_{-x} - 672x^4L_{-x} + 648\zeta_2L_{-x} - 2160x\zeta_2\,L_{-x} \\
& + 3456x^2\zeta_2L_{-x} - 2160x^3\zeta_2\,L_{-x} + 648x^4\zeta_2L_{-x} - 360L_{-x}^2 + 1044x\,L_{-x}^2 - 1458x^2L_{-x}^2 \\
& + 1044x^3\,L_{-x}^2 - 360x^4L_{-x}^2 + 108L_{-x}^3 - 216xL_{-x}^3 + 252x^2L_{-x}^3 - 144\,x^3L_{-x}^3 + 36x^4L_{-x}^3 \\
& + 3534L_{+x} - 5259xL_{+x} + 6309x^2L_{+x} - 2775x^3L_{+x} + 672x^4\,L_{+x} + 648\zeta_2L_{+x} - 1944x\zeta_2\,L_{+x} \\
& + 702x^2\zeta_2L_{+x} + 864x^3\zeta_2L_{+x} - 648x^4\,\zeta_2L_{+x} + 1080L_{-x}L_{+x} - 2106x\,L_{-x}L_{+x} + 1998x^2L_{-x}L_{+x} \\
& - 540x^3\,L_{-x}L_{+x} - 135x^2L_{-x}^2L_{+x} + 162\,x^3L_{-x}^2L_{+x} - 108x^4L_{-x}^2L_{+x} - 990\,L_{+x}^2 + 1656xL_{+x}^2 \\
& - 1512x^2L_{+x}^2 + 144x^3\,L_{+x}^2 + 360x^4L_{+x}^2 - 324L_{-x}L_{+x}^2 + 540x\,L_{-x}L_{+x}^2 - 216x^2L_{-x}\,L_{+x}^2 \\
& - 108x^3L_{-x}L_{+x}^2 + 108x^4L_{-x}L_{+x}^2 + 36\,L_{+x}^3 - 279xL_{+x}^3 + 198x^2L_{+x}^3 + 18x^3\,L_{+x}^3 - 36x^4L_{+x}^3 \\
& - 36\left(1-x+x^2\right)\left[-20\,\left(1-x+x^2\right) + 3x(-1+2x)L_{-x} - 6(-2+x)\,L_{+x}\right]\mathrm{Li}_2(1-x) \\
& + 72\left(1-x+x^2\right)\left[-10\left(1-x+x^2\right) + 3\left(3-2\,x+x^2\right)L_{+x}\right]\mathrm{Li}_2(x) + \Big[216\,L_{-x} - 324xL_{-x} \\
& + 324x^2\,L_{-x} - 108x^3\,L_{-x} - 216L_{+x} + 324xL_{+x} - 324x^2L_{+x} + 108x^3\,L_{+x}\Big]\mathrm{Li}_2\left(\frac{x}{x-1}\right) \\
& + \left[-108x + 324x^2 - 324x^3 + 216x^4\right]\mathrm{Li}_3(1-x) + \left[-216 + 432x - 648x^2 + 432x^3 - 216x^4\right] \times \\
& \times \mathrm{Li}_3(x) + \left[216 - 324x + 324x^2 - 108x^3\right]\mathrm{Li}_3\left(\frac{x}{x-1}\right)\Big\}
\end{aligned}$$

$$\begin{aligned}
& - \frac{1}{108(1-x+x^2)^2}\Big[818 - 1636x + 2454x^2 - 1636x^3 + 818x^4 + 216\zeta_2 - 432x\zeta_2 - 162x^2\zeta_2 \\
& + 540x^3\zeta_2 - 432x^4\zeta_2 + 120L_{-x} - 186xL_{-x} + 360x^2L_{-x} - 186x^3L_{-x} + 120x^4L_{-x} + 72L_{-x}^2 \\
& - 198xL_{-x}^2 + 270x^2L_{-x}^2 - 198x^3L_{-x}^2 + 72x^4L_{-x}^2 - 444L_{+x} + 672xL_{+x} - 846x^2L_{+x} \\
& + 402x^3L_{+x} - 120x^4L_{+x} - 216L_{-x}L_{+x} + 432xL_{-x}L_{+x} - 378x^2L_{-x}L_{+x} + 108x^3L_{-x}L_{+x} \\
& + 36L_{+x}^2 - 153xL_{+x}^2 + 135x^2L_{+x}^2 + 9x^3L_{+x}^2 - 72x^4L_{+x}^2 - 144\left(1-x+x^2\right)^2\mathrm{Li}_2(1-x) \\
& + 144\left(1-x+x^2\right)^2\mathrm{Li}_2(x)\Big]
\end{aligned}$$

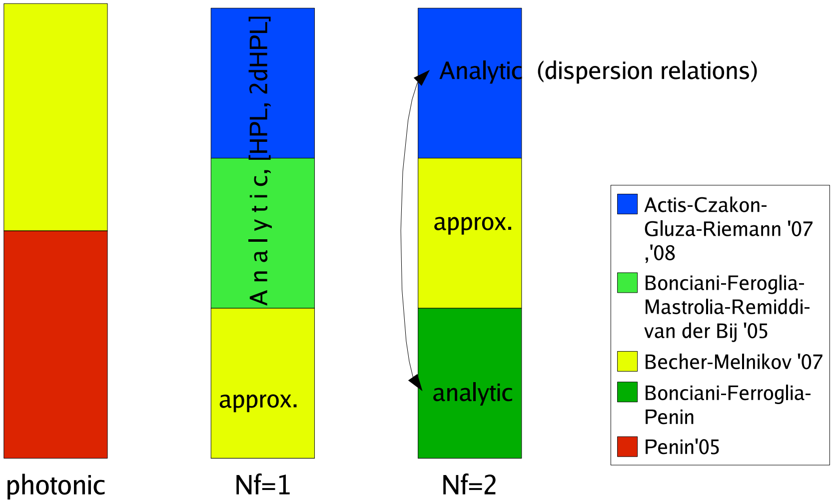
$$\Uparrow$$

$$\underbrace{\qquad}_{C_1}$$

[S.A.,Czakon,Gluza,Riemann '07]

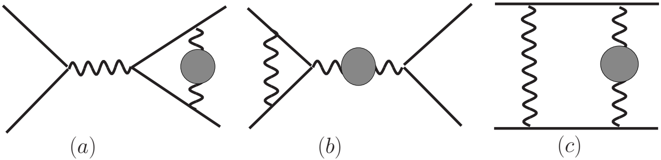
[Becher,Melnikov '07]

Present situation, virtual NNLO QED



Hadronic contributions

1. [ACGR, Physical Review Letters, 100, 2008]



Beyond $m_f^2 << s$

- KLOE $\sqrt{s} = 1$ GeV $< m_\tau$
- ILC $\sqrt{s} = 500$ GeV $\Rightarrow m_t < \sqrt{-t}, \sqrt{-u}$

  $40^0 < \theta < 140^0 \neq$ region for luminosity

$\Rightarrow$ Kinematical regions where expansions don't work

$\Rightarrow$ General method for including hadronic effects

Dispersion Relations

- Obtain fermionic corrections inserting Π_R in $\Delta_\gamma^{\mu\nu}$

$$\frac{g_{\mu\nu}}{q^2 + i\,\delta} \;\rightarrow\; \frac{g_{\mu\alpha}}{q^2 + i\,\delta}\,\left(q^2\, g^{\alpha\beta} - q^\alpha\, q^\beta\right)\; \Pi_R(q^2)\, \frac{g_{\beta\nu}}{q^2 + i\,\delta}$$

- Represent Π_R through a dispersion integral

$$\Pi_R(q^2) = -\frac{q^2}{\pi}\int_{4M^2}^{\infty} \frac{dz}{z}\, \frac{\mathrm{Im}\,\Pi(z)}{q^2 - z + i\,\delta}$$

Leptons, top (perturbative)

$$\mathrm{Im}\,\Pi_f(z) = -\left(\frac{\alpha}{\pi}\right) F_\epsilon \left(\frac{m_e^2}{m_f^2}\right)^{\epsilon} Q_f^2\, C_f \times$$

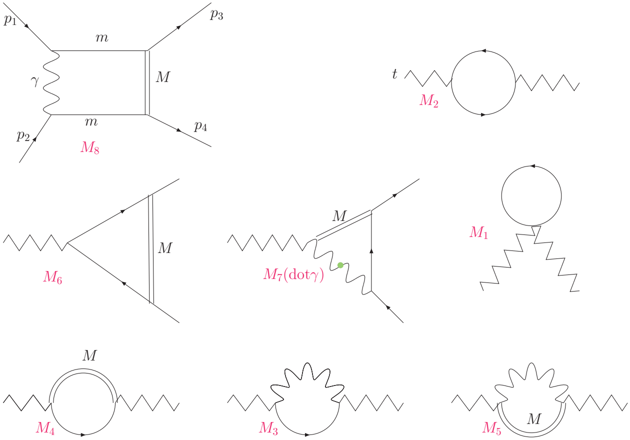
$$\times\,\theta\left(z - 4\,m_f^2\right)\,\frac{\pi}{3}\left\{\frac{\beta_f(z)}{2}\left[3 - \beta_f^2(z)\right]\right\}$$

Light quarks (non pert.)

Unitarity $\Rightarrow \mathrm{Im}\,\Pi_{\rm had}(z) = -\frac{\alpha}{3}\, R_{\rm had}(z)$

$$R_{\rm had}(z) = \frac{\sigma(\{e^+e^- \rightarrow \gamma^\star \rightarrow \text{hadrons}\}; z)}{(4\pi\alpha^2)/(3z)}$$

Effectively we must calculate following MIs



► again expanded from MB representations

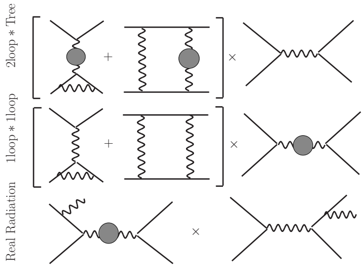
Box Master, M_8

$$\int \frac{d^D k}{(k^2 - z)\left[(k+p_3)^2 - m_e^2\right](k+p_3-p_1)^2\left[(k+p_3-p_1-p_2)^2 - m_e^2\right]}$$

$$\begin{aligned}
= \quad & \frac{1}{s\,(t-z)}\left\{\frac{1}{\epsilon}\left[\ln\!\left(-\frac{m_e^2}{t}\right) + \ln\!\left(-\frac{z}{s}\right) - \ln\!\left(-\frac{z}{t}\right)\right] - 2\,\zeta_2\right. \\
+ \quad & \ln\!\left(-\frac{m_e^2}{t}\right)\left[\frac{1}{2}\ln\!\left(-\frac{m_e^2}{t}\right) + \ln\!\left(-\frac{z}{s}\right) + \ln\!\left(-\frac{z}{t}\right) - 2\ln\!\left(1-\frac{z}{t}\right)\right] \\
- \quad & \frac{3}{2}\ln^2\!\left(-\frac{z}{t}\right) + \ln\!\left(-\frac{z}{s}\right)\ln\!\left(-\frac{z}{t}\right) \\
- \quad & 2\ln\!\left(1-\frac{z}{t}\right)\left[\ln\!\left(-\frac{z}{s}\right) - \ln\!\left(-\frac{z}{t}\right)\right] - \mathrm{Li}_2\left(1+\frac{z}{s}\right)\left.\right\} + \mathcal{O}(m_e^2).
\end{aligned}$$

IR finite results with boxes

The 4 direct and 4 crossed fermionic 2-loop box diagrams have to be combined with other diagrams for an IR-finite contribution:



► assembling all diagrams, the terms in $\ln(s/m_e^2)$ drop out and the total contribution of fermionic box diagrams is free of collinear divergencies

► a sensible, infrared safe cross-section contains the complete sum of all the single IR-divergent diagrams, or no one of them

Hadronic $\rightarrow$ leptonic, $m_\pi \rightarrow m_f$, plus change of kernels

$\sqrt{s}$ [GeV]	1	10	M_Z	500
boxes+red. e	193	5.73	0.1357	0.00673
μ	$<$ **1**	**0.42**	**0.0408**	**0.00288**
	–	0.08	0.0407	0.00288
τ	$<$ **1**	$<$ $\mathbf{10^{-2}}$	**0.0027**	**0.00088**
	–	–	-0.0096	0.00084
t	$<$ **1**	$<$ $\mathbf{10^{-2}}$	$<$ $\mathbf{10^{-4}}$	$<$ $\mathbf{10^{-5}}$
	–	–	–	–
had	$<$ **1**	**0.39**	**0.0877**	**0.00811**

Table: Differential cross sections in nanobarns at a scattering angle $\theta = 3^\circ$, in units of 10^2. net sum of infrared-sensitive corrections, including double boxes, dispersion approach (first line) and analytical expansion [ACGR:2007] (second line). When $m_f^2 > s, |t|, |u|$, the entry is suppressed. Parameters: $\omega = \sqrt{s}/2$, M_Z = 91.1876 GeV, m_t=172.5 GeV.

arXiv:0711.3847, version 1
calculations with R_{had} programs by Burkhardt

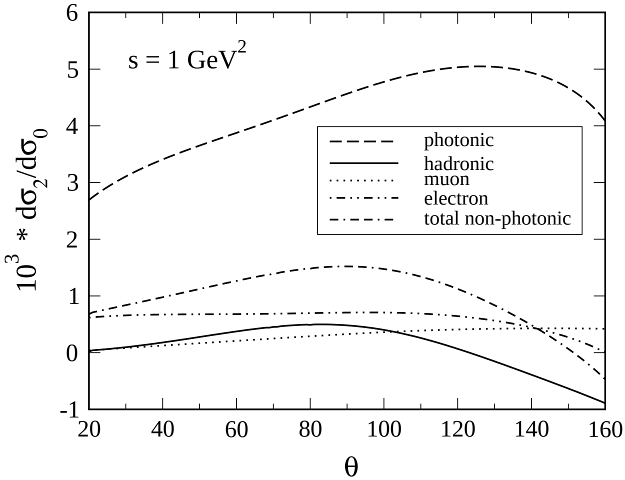
Numbers from Bonciani, Ferroglia, Penin, ’08

$\sqrt{s} = 1$ GeV

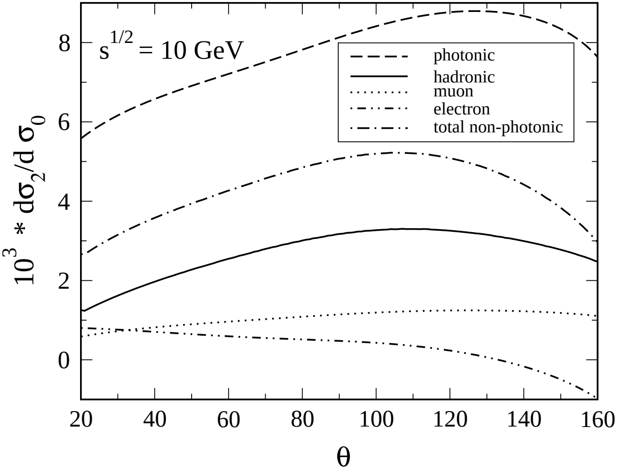
θ	e (10^{-4})	μ (10^{-4})	c (10^{-4})	τ (10^{-4})	b (10^{-4})
50°	17.341004	1.7972877	0.0622677	0.0264013	0.0010328
60°	18.407836	2.2267654	0.0861876	0.0367058	0.0014184
70°	19.438718	2.6504950	0.1086126	0.0465329	0.0018907
80°	20.465455	3.0655973	0.1253094	0.0540991	0.0022442
90°	21.463240	3.4581845	0.1321857	0.0576348	0.0024428
100°	22.366427	3.8070041	0.1268594	0.0560581	0.0024304
110°	23.099679	4.0922189	0.1098317	0.0495028	0.0022024
120°	23.605216	4.3030725	0.0843311	0.0392810	0.0018086
130°	23.847394	4.4392717	0.0549436	0.0273145	0.0013297

agreement (in all digits for μ)
and for $m_\tau = 1.7$ GeV [but not 1.777 GeV] :-))

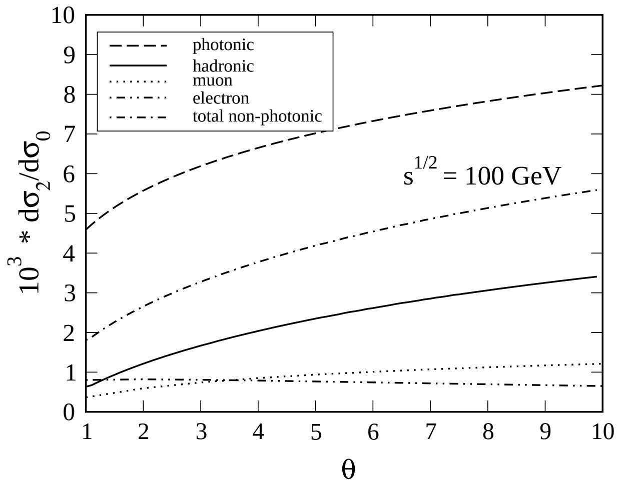
arXiv:0711.3847, version 2 → PRL



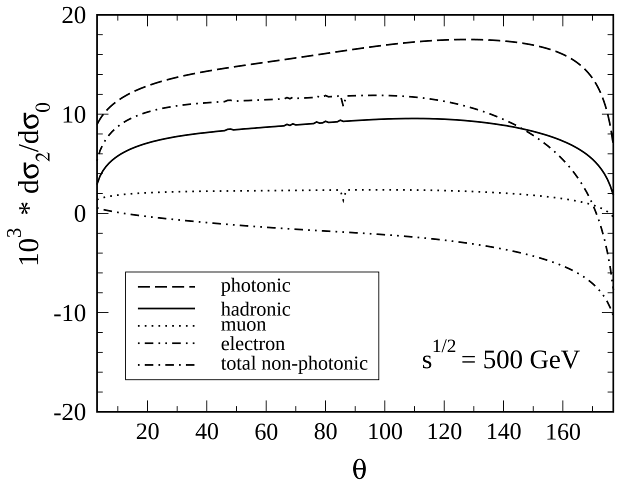
arXiv:0711.3847, version 2 $\rightarrow$ PRL



arXiv:0711.3847, version 2 $\longrightarrow$ PRL



arXiv:0711.3847, version 2 $\rightarrow$ PRL



Summary

- we are definitely in the LHC era, however, in the last 2 years NNLO QED Bhabha corrections have been extensively studied by different groups and they reach the level of several permille in the regions needed for measuring the luminosity
  $\Rightarrow$ it's good (in my opinion)
- their contributions are small, but non-negligible at the permille level
- the results have to be compared and interfaced with the available MC generators