News on Ambre and CSectors:

Numerical Evaluation Of Feynman Tensor Integrals In Euclidean Kinematical Region

Janusz Gluza, Katowice, Poland

in collaboration with **Krzysztof Kajda, Tord Riemann and Valery Yundin**

Loops and Legs, Wörlitz, 27 April 2010

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  $\rightarrow$ easy connection with Minkowskian space,
  J.G., T. Riemann, PoS RADCOR2007:007,2007

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- useful as a testing ground for relations obtained in reduction of tensor Feynman integrals

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- numerical checks of some relations where exact methods fail

- e.g. Bern, Czakon, Dixon, Kosower, Smirnov, PRD 2007

  "There is now significant evidence of a very simple structure in the planar limit. In particular, the planar contributions to the two-loop and three-loop four-gluon amplitudes have been shown to obey iterative relations ..."

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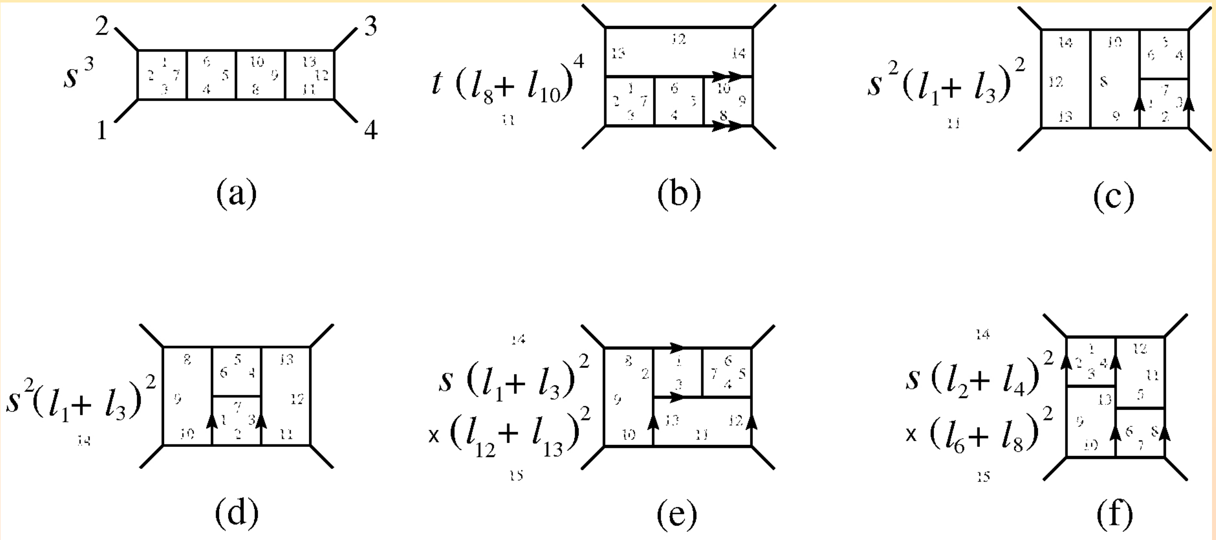
$$M_4^{(4)}(\rho;\epsilon) \quad = \quad \frac{1}{4}\Big[M_4^{(1)}(\rho;\epsilon)\Big]^4 - \Big[M_4^{(1)}(\rho;\epsilon)\Big]^2 M_4^{(2)}(\rho;\epsilon) + M_4^{(1)}(\rho;\epsilon) M_4^{(3)}(\rho;\epsilon) + \frac{1}{2}\Big[M_4^{(2)}(\rho;\epsilon)\Big]^2 + f^{(4)}(\epsilon)\, M_4^{(1)}(\rho;4\,\epsilon) + C^{(4)} + \mathcal{O}(\epsilon)$$



Methods of calculations

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- Mellin-Barnes (MB)

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- Mellin-Barnes (MB)
- sector decomposition (SD)

❖ AMBRE www

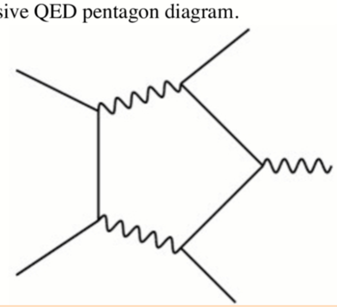
AMBRE - Automatic Mellin-Barnes REpresentation

arXiv: 0704.2423

J. Gluza, K. Kajda (Silesia U.) , T. Riemann (DESY, Zeuthen)

To download 'right click' and 'save target as'.

- **The package AMBRE.m, version 1.2**
  This version allows to generate M-B representations for tensor integrals containing not only scalar products of internal and external momenta, but also internal momenta with indices only. Additionally new options were added, among others it allows to generate representations without doing "X" integration (here we would like to thank Pierpaolo Mastrolia for this suggestion). Detailed description of new features is avaliable in the following examples:
  - description of new features: mathematica file
  - example of QED vertex with the following numerators: (k1.k1)^2, numerator with general external momenta, numerator without external momenta - example file
- **The package AMBREv1.1.m, version 1.1**
  This version allows to obtain MB-representations for direct products of Feynman integrals like e.g. tadpole*box, SE*vertex, etc.
  We thank Stefan Bekavac for indicating to the interest in this extension.
- **The package AMBREv1.0.m, version 1.0**
  This version is described in arXiv:0704.2423
  and Computer Physics Communications 177 (2007) 879, see some details from the CPC web page: here
- Kinematics generator for 4- 5- and 6- point functions with any external legs KinematicsGen.m with examples
- Tarball with examples given below examples.tar.gz
  - example1.nb, example2.nb - Massive QED pentagon diagram.



So far without possibility to calculate any tensor multiloop Feynman integrals (only one-loop cases)

- plus M. Czakon MB package (analytic continuation) MB tools

- Home
- Downloads
- Mailing list
- Tracker
- Wiki

MB Tools

This project is a collection of tools devoted to the evaluation of Mellin-Barnes integrals.

The project has been started by Michael Czakon; currently the web-page is also being updated by Alexander Smirnov.

The project is at the development stage, so expect more codes to appear here.

Currently the following codes can be downloaded:

- **MB.m** : version 1.2 of MB (last updated January 2nd, 2009) by Michal Czakon,
  the main collection of routines for the resolution of singularities and the numerical evaluation of Mellin-Barnes integrals;
  for details see hep-ph/0511200;
  the current version is documented in the **Manual** ;
  the distribution contains two example notebooks, MBexamples1.nb and MBexamples2.nb;
- **MBasymptotics.m** : a routine which expands Mellin-Barnes integrals in a small parameter by Michal Czakon;
  example usage is illustrated in MBasymptotics.nb;
- **MBresolve.m** : a tool by Alexander Smirnov and Vladimir Smirnov realizing another strategy of resolving singularities of Mellin-Barnes integrals. This code should be loaded together with **MB.m** since it uses some of its routines. For details see arXiv:0901.0386
- **AMBRE.m** : a tool by Janusz Gluza, Krzysztof Kajda and Tord Riemann for constructing Mellin-Barnes representations. It works both for planar multiloop scalar and one-loop tensor Feynman integrals. This is version 1.2, for previous versions and detailed description of the package with examples see the **home page** . The program is described in arXiv:0704.2423 and Computer Physics Communications 177 (2007) 879.
- **barnesroutines.m** : a tool by David Kosower for automatic application of the first and second Barnes lemmas on lists of multiple Mellin-Barnes integrals. An example notebook is included.

The numerical integration routines used by MB require the following libraries to be installed, either in the current working directory, or in the global repository for libraries (e.g. /usr/local/lib)

Tensors

General form $(T(k) = 1, k_l^\mu, k_l^\mu k_n^\nu, \ldots)$

$$G_L[T(k)] = \frac{1}{(i\pi^{d/2})^L} \int \frac{d^dk_1 \ldots d^dk_L \;\; T(k)}{(q_1^2 - m_1^2)^{\nu_1} \ldots (q_i^2 - m_i^2)^{\nu_j} \ldots (q_N^2 - m_N^2)^{\nu_N}}.$$

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After Feynman parametrization

$$\begin{aligned} G_L[T(k)] &= \frac{(-1)^{N_\nu}}{\Gamma(\nu_1)\ldots\Gamma(\nu_N)} \int \prod_{j=1}^{n} dx_j x_j^{\nu_j - 1} \delta\left(1 - \sum_{i=1}^{n} x_i\right) \\ &\times \sum_{r \leq m} \frac{\Gamma\left(N_\nu - \frac{d}{2}L - \frac{r}{2}\right)}{(-2)^{\frac{r}{2}}} \frac{U^{N_\nu - \frac{d}{2}(L+1) - m}}{F^{N_\nu - \frac{d}{2}L - \frac{r}{2}}} \left\{\mathcal{A}_r P^{m-r}\right\}^{[\mu_1, \ldots, \mu_m]} \end{aligned}$$

The object: $\{\mathcal{A}_r P^{m-r}\}^{[\mu_1,\ldots,\mu_m]}$ is used to introduce tensor structure:

- m=2

$$\sum_{r\leq 2}\left\{\mathcal{A}_r P^{2-r}\right\}^{[\mu_1\mu_2]} = \left\{A_0P^2 + A_1P^1 + A_2P^0\right\}^{[\mu_1\mu_2]}$$

$$= P^{\mu_1}P^{\mu_2} + \tilde{g}^{\mu_1\mu_2}$$

- m=3

$$\sum_{r\leq 3}\left\{\mathcal{A}_r P^{3-r}\right\}^{[\mu_1\mu_2\mu_3]} = \left\{A_0P^3 + A_1P^2 + A_2P^1 + A_3P^0\right\}^{[\mu_1\mu_2\mu_3]}$$

$$= P^{\mu_1}P^{\mu_2}P^{\mu_3} + \tilde{g}^{\mu_1\mu_2}P^{\mu_3} + \tilde{g}^{\mu_2\mu_3}P^{\mu_1} + \tilde{g}^{\mu_3\mu_1}P^{\mu_2}$$

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A_0, P^0 is one. A_r is zero for r odd, and $\mathcal{A}_r = \tilde{g}^{[\mu_1\mu_2}\cdots\tilde{g}^{\mu_{r-1}\mu_r]}$ for r even.

a little bit more...

P^{μ_i} and $\tilde{g}^{\mu_i \mu_j}$...

$$P^{\mu_i} \to \sum_l [\tilde{M}_{al} Q_l]_{\mu_i}$$

$$\tilde{g}^{\mu_i \mu_j} \to (\tilde{M}^{-1})_{ab} g^{\mu_i \mu_j}$$

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$$\sum_{i=1}^{n} P_i x_i = \sum_{i=1}^{n} (q_i^2 - m_i^2) x_i = \sum_{i,j=1}^{L} k_i^T M_{ij} k_j - 2 \sum_{j=1}^{L} k_j^T Q_j + J,$$

$$F = -\mathrm{det}(M) J + Q\tilde{M}Q, \quad \tilde{M} = \mathrm{det}(M) M^{-1}.$$

F polynomial and Mellin-Barnes

$$
\begin{aligned}
\frac{1}{(A_1+\ldots+A_n)^\lambda} &= \frac{1}{\Gamma(\lambda)}\frac{1}{(2\pi i)^{n-1}}\int_{c-i\infty}^{c+i\infty}\ldots\int_{c-i\infty}^{c+i\infty} dz_2\ldots dz_n \prod_{i=2}^{n} A_i^{z_i} \\
&\times \quad A_1^{-\lambda-z_2-\ldots-z_n}\Gamma(\lambda+z_2+\ldots+z_n)\prod_{i=2}^{n}\Gamma(-z_i)
\end{aligned}
$$

n terms leads to $n-1$ complex integrals.

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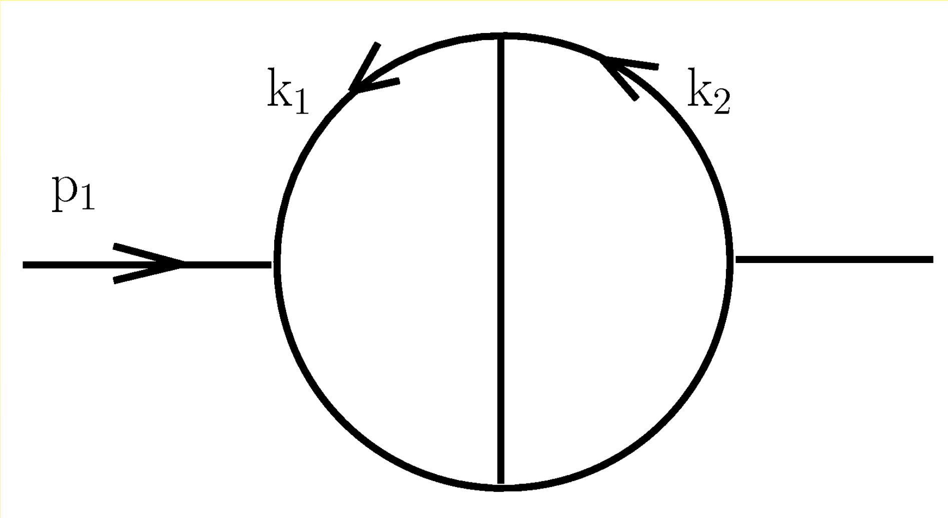
Finally, integration over Feynman parameters

$$\int_0^1 \prod_{i=1}^{n} dx_j x_j^{a_j-1}\delta\Big(1-\sum_{i=1}^{n} x_i\Big) = \frac{\Gamma(a_1)\ldots\Gamma(a_n)}{\Gamma(a_1+\cdots+a_n)}$$

SE with numerator



$$\int \frac{(k_1 \cdot p)(k_1 \cdot p)(k_2 \cdot p)}{[k_1^2]^{n_1}[(k_2-k_1)^2]^{n_2}[(k_1+p)^2]^{n_3}[k_2^2]^{n_4}[(k_2+p)^2]^{n_5}} d^d k_1 d^d k_2.$$

$$F = -[k_2]^2 x_1 x_2 - s x_1 x_3 - [(k_2+p)^2] x_2 x_3$$

Tensor structure

$$
\begin{aligned}
P^{\mu_1}P^{\mu_2} + \tilde{g}^{\mu_1\mu_2} &\rightarrow Q^{\mu_1}Q^{\mu_2} + g^{\mu_1\mu_2} \\
&\rightarrow (k_2^{\mu_1}x_2 - p^{\mu_1}x_3)(k_2^{\mu_2}x_2 - p^{\mu_2}x_3) + g^{\mu_1\mu_2} \\
&\rightarrow \{k_2^{\mu_1}k_2^{\mu_2}x_2^2, -k_2^{\mu_2}p^{\mu_1}x_2x_3, k_2^{\mu_1}p^{\mu_2}x_2x_3, p^{\mu_1}p^{\mu_2}x_3^2, g^{\mu_1\mu_2}\}
\end{aligned}
$$

Tensor structure

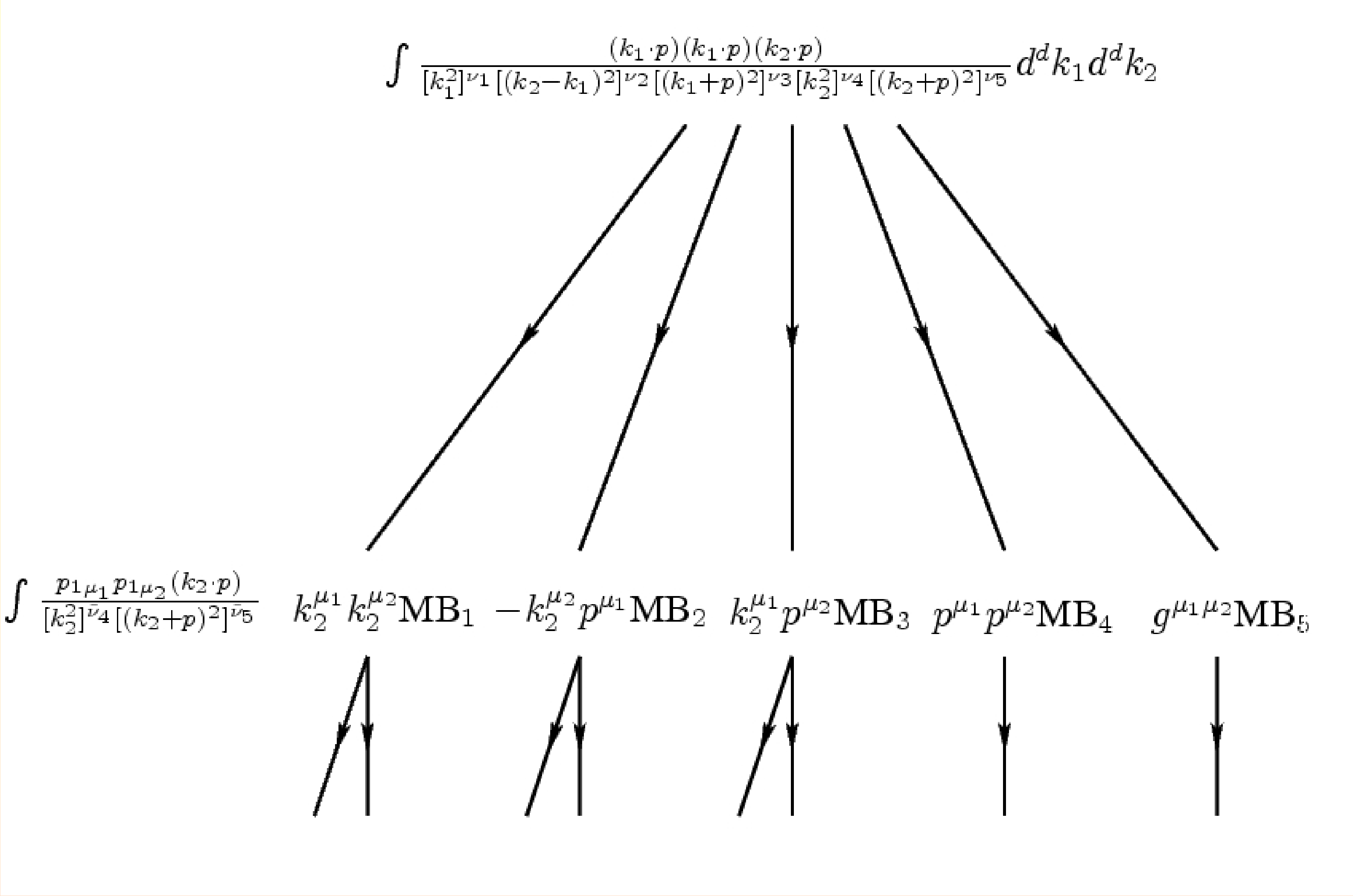
$$\begin{aligned} P^{\mu_1}P^{\mu_2}+\tilde{g}^{\mu_1\mu_2} &\rightarrow Q^{\mu_1}Q^{\mu_2}+g^{\mu_1\mu_2} \\ &\rightarrow (k_2^{\mu_1}x_2-p^{\mu_1}x_3)(k_2^{\mu_2}x_2-p^{\mu_2}x_3)+g^{\mu_1\mu_2} \\ &\rightarrow \{k_2^{\mu_1}k_2^{\mu_2}x_2^2, -k_2^{\mu_2}p^{\mu_1}x_2x_3, k_2^{\mu_1}p^{\mu_2}x_2x_3, p^{\mu_1}p^{\mu_2}x_3^2, g^{\mu_1\mu_2}\} \end{aligned}$$

Usually a rank of a given integral in the next step of iteration will include higher rank tensors than the original object

$$\int \frac{p_{1\mu_1}p_{1\mu_2}(k_2\cdot p)}{[k_2^2]^{-z_1}[(k_2+p)^2]^{-3+\epsilon+n_1+n_2+n_3+z_1+z_2}}$$

$$\times \quad \{k_2^{\mu_1}k_2^{\mu_2}\mathrm{MB}_1, -k_2^{\mu_2}p^{\mu_1}\mathrm{MB}_2, k_2^{\mu_1}p^{\mu_2}\mathrm{MB}_3, p^{\mu_1}p^{\mu_2}\mathrm{MB}_4, g^{\mu_1\mu_2}\mathrm{MB}_5\}d^dk_2,$$

$$\begin{aligned} \mathrm{MB}_1 &= ((-1)^{2-\epsilon-z_2}(-s)^{z_2}\Gamma(2-\epsilon-n_1-n_2-z_1)\Gamma(-z_1)\Gamma(4-\epsilon-n_1-n_3-z_2) \\ &\times \Gamma(-z_2)\Gamma(n_1+z_1+z_2)\Gamma(-2+\epsilon+n_1+n_2+n_3+z_1+z_2))/ \\ &\times (\Gamma(n_1)\Gamma(n_2)\Gamma(6-2\epsilon-n_1-n_2-n_3)\Gamma(n_3)). \end{aligned}$$


$\int \frac{(k_1\cdot p)(k_1\cdot p)(k_2\cdot p)}{[k_1^2]^{\nu_1}[(k_2-k_1)^2]^{\nu_2}[(k_1+p)^2]^{\nu_3}[k_2^2]^{\nu_4}[(k_2+p)^2]^{\nu_5}} d^dk_1 d^dk_2$
$\int \frac{p_{1\mu_1} p_{1\mu_2} (k_2\cdot p)}{[k_2^2]^{\bar{\nu}_4}[(k_2+p)^2]^{\bar{\nu}_5}}$
$k_2^{\mu_1} k_2^{\mu_2} \mathrm{MB}_1$
$-k_2^{\mu_2} p^{\mu_1} \mathrm{MB}_2$
$k_2^{\mu_1} p^{\mu_2} \mathrm{MB}_3$
$p^{\mu_1} p^{\mu_2} \mathrm{MB}_4$
$g^{\mu_1\mu_2} \mathrm{MB}_5$

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```
<< AMBREn.m
invariants = {p1^2->s};
MBrepr[{k1*p1,k1*p1,k2*p1},{PR[k1,0,n1]*PR[k2,0,n2]*
       PR[k2-k1,0,n3]*PR[k1+p1,0,n4]*PR[k2+p1,0,n5]},{k1,k2}]
```

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```

Output:[1]

```
  repr = {-(((-1)^(n1+n2+n3+n4+n5)*(-s)^(4-2*eps-n1-n2-n3-n4-n5)*s^3
      Gamma[2-eps-n1-n3-z1]*Gamma[-z1]*Gamma[5-eps-n2+z1]
 *Gamma[4-eps-n1-n4-z2]*Gamma[4-2*eps-n1-n3-n4-n5-z1-z2]
 *Gamma[-z2]*Gamma[-4+2*eps+n1+n2+n3+n4+n5+z2]*Gamma[n1+z1+z2]
 *Gamma[-2+eps+n1+n3+n4+z1+z2])/(Gamma[n1]*Gamma[n3]
 *Gamma[6-2*eps-n1-n3-n4]*Gamma[n4]*Gamma[n2-z1]
 *Gamma[9-3*eps-n1-n2-n3-n4-n5-z2]*Gamma[-2+eps+n1+n3+n4+n5+z1+z2])),...}
```

[1] User can use Options to control output and its steps
Options[MBrepr] = Text − > True, BarnesLemma1 − > True, BarnesLemma2 − > False;

MBnum.m

- for multiloops usually many MB integrals, we need analytic continuation for them in ϵ parameter, sometimes even in additional parameter e.g. connected with a power of first propagator: $n_1 = 1$ then $n_1 \quad \rightarrow \quad n_1 = 1 + \eta$

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- ```
  MBnum[repr, 0, {m -> 1, s -> -5, t -> -7},
    {n1 -> 1, n2 -> 1, n3 -> 1, n4 -> 1, n5 -> 1, n6 -> 1, n7 -> 1,
    n8 -> 1}, 2] // AbsoluteTiming

  ETA's will be aplied on positions: {}

  1. Calculating 'no eta' parts...

     Running MBcontinue...
     Running MBexpand...
     Running MBintegrate...
  ```
  ```

```
2. Calculating 'eta' parts...
   No 'eta' parts found!!!

Out[]= {47.559004, -0.1034149 -0.0228571/eps^4 + 0.0831877/eps^3
        - 0.0096574/eps^2 - 0.109073/eps}
```

Sector decomposition

"Sector decomposition is a constructive method to isolate divergences from parameter integrals...",

a comprehensive review by G. Heinrich, arXiv:0803.4177

Sector-decomposition by Ch.Bogner and S.Weinzierl (arXiv:0709.4092 [hep-ph]) calculates basic parts needed for the tensor structure[2]:

$$\int\limits_{x_j \geq 0} d^n x \, \delta(1 - \sum_{i=1}^{n} x_i) \left(\prod_{i=1}^{n} x_i^{a_i + \epsilon b_i} \right) \prod_{j=1}^{r} \left[P_j(x) \right]^{c_j + \epsilon d_j} .$$

$$\begin{aligned} G_L[T(k)] \quad &= \quad \frac{(-1)^{N\nu}}{\Gamma(\nu_1)\ldots\Gamma(\nu_N)} \int \prod_{j=1}^{n} dx_j x_j^{\nu_j - 1} \delta\left(1 - \sum_{i=1}^{n} x_i\right) \\ &\times \quad \sum_{r \leq m} \frac{\Gamma\left(N_\nu - \frac{d}{2}L - \frac{r}{2}\right)}{(-2)^{\frac{r}{2}}} \frac{U^{N\nu - \frac{d}{2}(L+1) - m}}{F^{N\nu - \frac{d}{2}L - \frac{r}{2}}} \left\{ \mathcal{A}_r P^{m-r} \right\}^{[\mu_1, \ldots, \mu_m]} \end{aligned}$$

[2]another public program is FIESTA by Tentyukov and Smirnov

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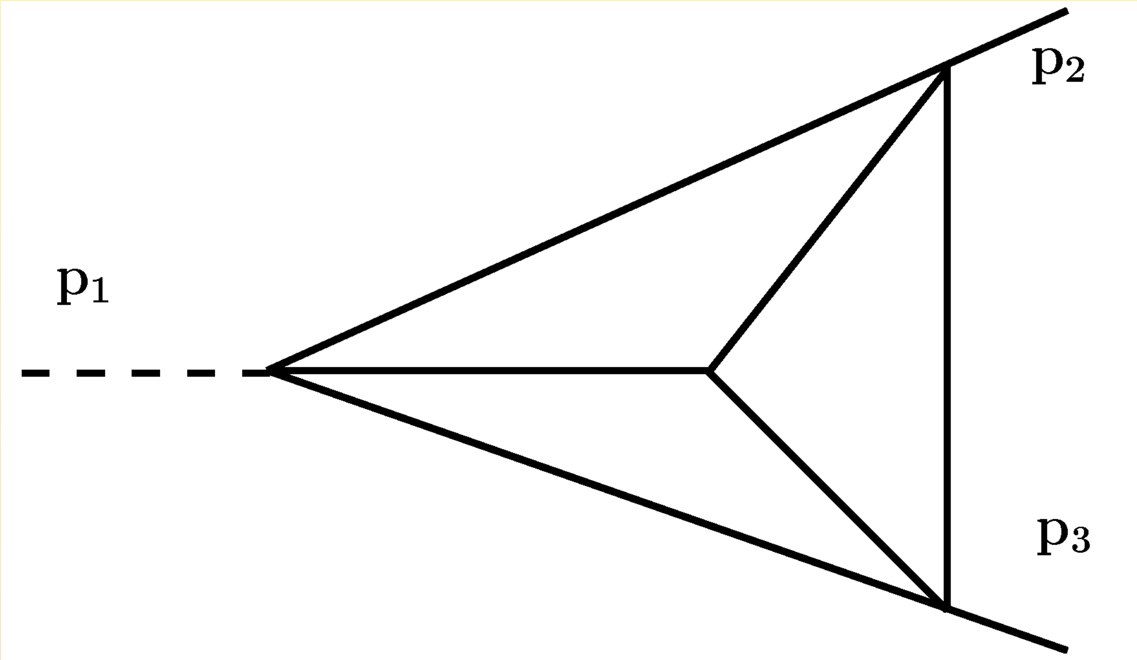
- is a MATHEMATICA interface linked with GiNaC libraries of "sector decomposition" by Bogner and Weinzierl
- can build m-rank tensor structure for L-loop integrals
- process of numerical calculation of integrals is fully automatic

3 loop vertex of rank 3

$$\int d^dk_1 d^dk_2 d^dk_3 \frac{k_2 \cdot p_2 \; k_3 \cdot p_1 \; k_3 \cdot p_1}{k_1^6 k_2^2 k_3^2 (k_2+k_3+p_2)^2 (k_1-k_3)^2 (k_1+k_2-p_1+p_2)^2}$$



A part in the Mathematica environment

```
<< CSectors.m;
x=-11;
invariants={p1*p2->1/2*x,p1*p3->1/2*x,p2*p3->1/2*x,p1^2->x,
            p2^2->0,p3^2->0};
DoSectors[{k2*p2,k3*p1,k3*p1},{PR[k2+k3+p2,0,1]*PR[k1-k3,0,1]*
          PR[k3,0,1]*PR[k1+k2-p1+p2,0,1]*PR[k1,0,1]*PR[k2,0,3]},
          {k1,k2,k3}][-5,0];
```

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            p2^2->0,p3^2->0};
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          PR[k3,0,1]*PR[k1+k2-p1+p2,0,1]*PR[k1,0,1]*PR[k2,0,3]},
          {k1,k2,k3}][-5,0];
```

Output[3]:

```
CSectors by K.Kajda and V.Yundin ver:1.0
last modified 25.04.2010

Using strategy C
```

[3]Speed and efficiency of calculations depends on the algorithm, defined as strategies A-D by Bogner and Weinzierl

```
U & F polynomials:

   U  =  x4 (x5 x6 + x3 (x5 + x6)) + x1 (x4 x5 + x2
         (x3 + x4 + x5) + x4 x6 + x5 x6 + x3 (x5 + x6)) + x2
         ((x4 + x5) x6 + x3 (x4 + x5 + x6))
   F  = 11 (x3 (x2 + x4) + x1 (x3 + x4)) x5 x6

   Q11 = ... (a few lines)
   Q12 = ... (a few lines)

   Q21 = 121*x2*x3^2*x6^2/4

   Q22 = ... (a few lines)
```

Q's are generated terms by the tensor structure $\{\mathcal{A}_r P^{m-r}\}^{[\mu_1,\ldots,\mu_m]}$

internal work

```
Generating c++ source...Int11...Int12...Int21...Int22...done
Compiling source code...Int11...Int12...Int21...Int22...done
Running binary file.....Int11...Int12...Int21...Int22...done
```

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```

and the result is

```
Result=
{-7.622574999999999 - 0.0260407/eps^4
 + 0.049527000000000015/eps^3 - 0.4168788/eps^2 + 0.56955/eps,
 {1.3667737639753552, 2.85804370185272*^-6/eps^4,
  0.00009220935574625821/eps^3, 0.0004811810295896961/eps^2,
  0.006549529654501916/eps}}
```

Numerics for the massless and massive double planar box B1.

massless	AMBRE and MB	CSectors, X-strat.
ϵ^0	$-0.1034 \pm 6 \cdot 10^{-6}$	-0.1035 ± 0.0002
ϵ^{-1}	-0.10907	-0.10915 ± 0.00008
ϵ^{-2}	-0.00966	-0.00966 ± 0.00001
ϵ^{-3}	0.083188	0.083191 ± 0.000005
ϵ^{-4}	-0.022857	$-0.0228574 \pm 1 \cdot 10^{-6}$
T [s]	28	1712
$s = -5$, $t = -7$		

massive	AMBRE and MB	CSectors, C-strat.
ϵ^0	0.2246	0.2246 ± 0.0001
ϵ^{-1}	0.06359	0.06357 ± 0.00003
ϵ^{-2}	-0.023524	$-0.023524 \pm 4 \cdot 10^{-6}$
T [s]	50	345
$s = -5$, $t = -7$, $m = 1$		

- For massless cases, e.g. two-loop 4 point functions, sector decomposition method needs a lot of RAM memory (a few GB is not an exception). Then numerical values are easier to be found for integrals using MB method (especially when used with barnesroutines.m by D. Kosower, which can simplify dimensionality of an integral substantially).

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- Note different strategies used for efficient calculation within CSectors

Cross-checks

- iterative way: reduction of numerators by cancelations with propagators

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- comparison between results by MB and SD methods

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- I report on two user friendly programs AMBRE (should be used e.g. with MB.m by M. Czakon) and CSectors.m (should be used with the Ginac sector decomposition by Bogner and Weinzierl) which are able to calculate numerically tensor multiloop Feynman integrals in the Euclidean region

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- due to RAM memory and time of calculation, the programs are efficient for tensor two loop massive and massless boxes or 3-loop massive and massless vertices

- the programs can be complementary to each other

www will be launched soon

CSectors - numerical calculation of multiloop tensor integrals in Euclidean region by sector decomposition

arXiv: xxxx.yyyy

J. Gluza, K. Kajda (Silesia U.) , T. Riemann, V. Yundin (DESY, Zeuthen)

See here (Mellin-Barnes) for an alternative way of numerical calculation of Feynman Integrals in Euclidean region.

To download 'right click' and 'save target as'.

- **The package CSectors.m, version 1.0**

  The package compute numerically the Laurent expansion of (divergent) multi-loop tensor Feynman integrals. It generates appropriate files in an automatic way and links them with the basic sector decomposition program by Bogner and Weinzierl webpage. See there for description and download of the package altogether with Ginac. Detailed description how to define integrals for numerical calculations can be found in the paper arXiv: xxxx.yyyy and the following examples:

  - description of new features: mathematica file

Tarball with examples given below examples.tar.gz. Some of examples below correspond to the examples calculated by Mellin-Barnes on the webpage here , additional examples here include non-planar cases. All include some numerators.

- ex1_pentagon.sh, output_ex1_pentagon.sh - Massive QED pentagon diagram.