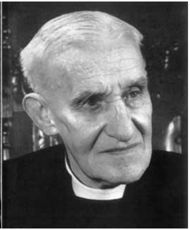
Mellin and Barnes meet in Minkowskian kinematics - new prospects on numerical MB methods - and applications

Mellin, Robert, Hjalmar, 1854-1933



Barnes, Ernest, William, 1874-1953

Janusz Gluza, U. Silesia, Katowice

Epiphany, Cracow 2017

Goals of the talk

Steady progress in particle physics needs
new ideas and crafting of ever-changing
theoretical tools and techniques of calculations.

- True enough for numerical tools:
  - SM precise studies - more "loops and legs"
  - NP - NLO level

Here: recent progress in the **MB numerical method** and potential applications **beyond 1-loop level**.

The talk based on collaborations, for details see:

1. **Ievgen Dubovyk, Ayres Freitas, JG,Tord Riemann, Johann Usovitsch**
   "The two-loop electroweak bosonic corrections to $\sin^2\theta_{\rm eff}^{\rm b}$"
   Phys.Lett. B762 (2016) 184
   - TR LL16 talk, PoS LL2016 (2016) 075:
     "30 years, some 700 integrals, and 1 dessert, or:
     Electroweak two-loop corrections to the $Z\bar{b}b$ vertex", arXiv:1610.07059;
   - JG LL16 talk, PoS LL2016 (2016) 034:
     "Numerical integration of massive two-loop Mellin-Barnes integrals in Minkowskian regions",
     arXiv:1607.07538

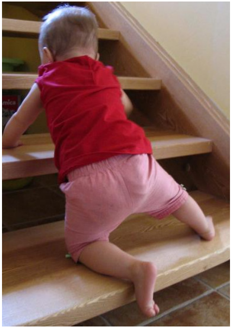
   First applications

2. JG, Tomasz Jeliński, David A. Kosower,
   "Efficient Evaluation of Massive Mellin–Barnes Integrals",
   e-Print: arXiv:1609.09111

Outline

Basic problem: Steping up from Euclidean to direct calculation in Minkowskian kinematics

$$\frac{1}{(-)p^2 - m^2} \longrightarrow singularities \longrightarrow \frac{1}{(-)p^2 - m^2 + i\delta^1}$$



[1] "One of the most remarkable discoveries in elementary particle physics has been that of the existence of the complex plane" - Julian Schwinger

Solution(s) for one-loop amplitudes are known

Independently of the given one-loop process, amplitudes are decomposed into basis of scalar integrals: boxes, triangles, bubbles and tadpoles

$$\text{Amplitude} = \sum_{j \in \text{Basis}} \text{c}_j * \text{Integral}_j + \text{Rational}$$

$$\text{Integral}_j = \int d^{4-2\epsilon}\ell \frac{1}{D_1 D_2 ... D_n}.$$

- Kallen, Toll (1965): triangles (n=3) → bubbles (in 2 dim)
- Melrose (1965, later van Neerven and Vermaseren): pentagon (n=5) → boxes (in 4 dim),
- Lorentz invariance + Passarino and Veltman: tensor n-PF → m-PF scalar integrals ($m \leq n$)
- *improved* tensor decomposition (Denner, Dittmaier, Fleischer, Riemann, Yundin,...)

Libraries: LoopTools (FF), OneLoop, QCDLoop.

NLO Automations

Over the last decade or so modern methods of

- on-shell recursion relations (Britto, Cachazo, Feng, Witten,...)

and

- unitarity methods (Bern, Dixon, Kosower, ..., Ossola, Pittau, Papadopoulos, ..., Badger,.....)

overtaken to a large extent traditional Feynman diagrammatic approach, including one-loop calculations.

Knowledge of (scalar) basis and their analytic structure allowed to focus and find coefficients of reductions (OPP, Kosower, ..., Mastrolia,...), integrand reduction techniques (Ellis, Giele, Kunszt,Melnikov, Tramontano, Heinrich, Reiter,...)

Altogether - a bunch of automatic packages: FeynArts, BlackHat, Golem/Samurai, GoSam, Helac-NLO, MadGraph@NLO, Collier, PJFRY, ...

Present target: two loops

Still, a general 2-loop basis does not worked out.

1. Laporta algorithm, reduction to Master Integrals using IBP relations (Chetyrkin-Tkachov)

$$0 = \prod_{i=1}^{L} \left(\int \frac{d^d \ell_i}{(2\pi)^d} \right) \frac{\partial}{\partial \ell_j} \cdot \left(\frac{v^{(j)}}{D_1(\ell_1, \ldots \ell_L) \cdots D_m(\ell_1, \ldots \ell_L)} \right)$$

2. Automation through a public software AIR, FIRE, Reduze, (plus IdSolver, etc)
3. Finite number of Master Integrals must be solved (e.g. by DEqs. method: Remiddi, Caffo, Czyż, recent progress: Henn 2013, Prausa 2017).

Direct numerical integrations in Minkowskian regions >NLO

Sector decomposition (SD)

FIESTA 3 [A.V.Smirnov, 2014]

SecDec 2 [S. Borowka, G. Heinrich, 2012]

SecDec 3 [S. Borowka, G. Heinrich, P. Jones, M. Kerner, J. Schlenk, T. Zirke, 2013]

- NICODEMOS, ver 2.0 [A. Freitas]

Now: the MB method

Two steps (automatic construction and numerical evaluation):

1. AMBRE [K.Kajda (planar, ver.2.2), E.Dubovyk (non-planar,ver3.0)]-PlanarityTest[K.Bielas, E.Dubovyk]
2. MBnumerics [J. Usovitsch, E. Dubovyk] - a completely new software !

MB used so far, some examples

- Evaluation of MIs (Tausk, Smirnov, ...)
- Bhabha massive QED 2-loop (M. Czakon, JG, T. Riemann, S. Actis) (MB & expansions), (MB & dispersion relations)

- "On the **Numerical** Evaluation of Loop Integrals With Mellin-Barnes Representations",
  Ayres Freitas, Yi-Cheng Huang, JHEP, 2010
- "Angular integrals in d dimensions",
  Gabor Somogyi, J.Math.Phys, 2011
- "Soft triple-real radiation for Higgs production at N3LO",
  C. Anastasiou, C. Duhr, F. Dulat, B. Mistlberger, JHEP, 2013
- "Evaluating multi-loop Feynman diagrams with infrared and threshold singularities **numerically**",
  C. Anastasiou, S. Beerli, A. Daleo, JHEP, 2007

Step 1

Construction of MB integrals

http://us.edu.pl/∼gluza/ambre/

Mellin-Barnes representations in HEP - method

- "Om definita integraler", R. H. Mellin, Acta Soc. Sci. Fenn. 20(7), 1 (1895), "The theory of the gamma function", E. W. Barnes Messenger Math. 29(2), 64 (1900).

$$mathematics \longrightarrow \frac{1}{(A+B)^{\lambda}} = \frac{1}{\Gamma(\lambda)} \frac{1}{2\pi i} \int_{-i\infty}^{+i\infty} dz \Gamma(\lambda+z)\Gamma(-z) \frac{B^z}{A^{\lambda+z}}$$

$$physics \longrightarrow \frac{1}{(p^2-m^2)^{a}} = \frac{1}{\Gamma(a)} \frac{1}{2\pi i} \int_{-i\infty}^{+i\infty} dz \Gamma(a+z)\Gamma(-z) \frac{(m^2)^z}{(p^2)^{a+z}}$$

It is recursive $\Longrightarrow$ multidimensional complex integrals.

$$\int_{-\frac{1}{3}-i\infty}^{-\frac{1}{3}+i\infty} dz_1 \int_{-\frac{2}{3}-i\infty}^{-\frac{2}{3}+i\infty} dz_2 \left(\frac{-s}{M_Z^2}\right)^{-z_1} \frac{\Gamma[-z_1]^3\Gamma[1+z_1]\Gamma[z_1-z_2]\Gamma[-z_2]^3\Gamma[1+z_2]\Gamma[1-z_1+z_2]}{s \;\; \Gamma[1-z_1]^2\Gamma[-z_1-z_2]\Gamma[1+z_1-z_2]}$$

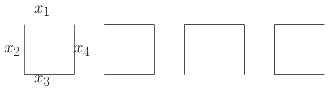
Overlaped integrals

Multiloop Feynman diagrams, general MB integrals

$$\frac{1}{D_1^{n_1} D_2^{n_2} \dots D_N^{n_N}} \to \int \prod_{j=1}^{N} dx_j \; x_j^{n_j - 1} \delta(1 - \sum_{i=1}^{N} x_i) \frac{U(x)^{N_\nu - d(L+1)/2}}{F(x)^{N_\nu - dL/2}}$$

$$N_\nu = n_1 + \dots + n_N$$

The functions U and F are called graph or Symanzik polynomials.

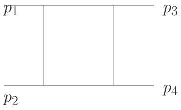


Trees contributing to the polynomial U for the square diagram

$$\mathbf{U} \quad = \quad \mathbf{x_1 + x_2 + x_3 + x_4} \quad ! \; \textit{1-loop} \longrightarrow 1$$

$$\mathbf{F} \quad = \quad \mathbf{t \cdot x_1 x_3 + s \cdot x_2 x_4}$$

2 – trees contributing to the polynomial F for the square diagram



Cuts of internal lines such that:

- U: (i) every vertex is still connected to every other vertex by a sequence of uncut lines; (ii) no further cuts without violating (i)
- F: (iii) divide the graph into two disjoint parts such that within each part (i) and (ii) are obeyed and such that at least one external momentum line is connected to each part;

Dimension of MB integrals depends on factorizations of F and U!

Step 2

Numerics of MB integrals

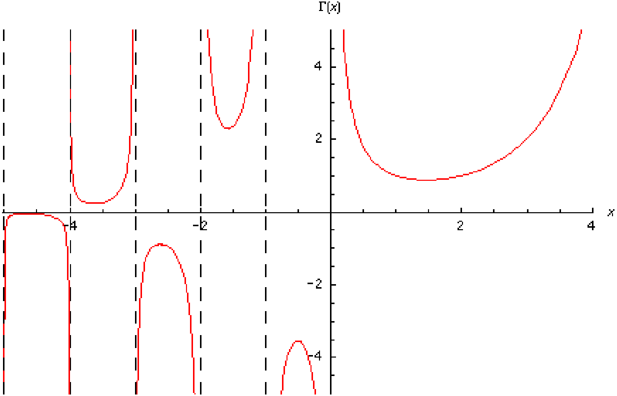
http://mbtools.hepforge.org/

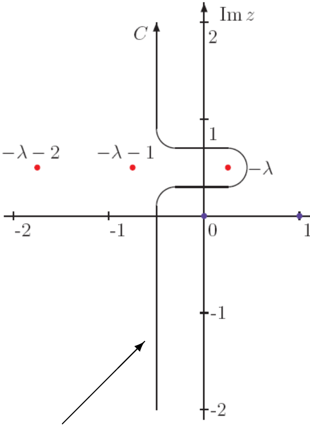
Gamma function: Singularities in the complex plane

$$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt$$

$$\int dz \Gamma[z+\lambda]$$

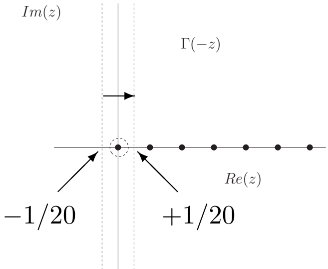
SINGULARITIES REGULAR





Contours: shifts, deformations

```
(* shifting contours *)
In[203]:=
sim = Gamma[-z]
Out[203]=
Gamma[-z]
In[227]:=
Sum[-Residue[Gamma[-z], {z, n}], {n, 0, 100}] // N
Out[227]=
0.367879
In[226]:=
n1 = NIntegrate[
  1 / (2 Pi) sim /. z → -1 / 20 + I y, {y, -10, 10}]
Out[226]=
0.367879 + 0. ⅈ
In[230]:=
n2 = NIntegrate[
  1 / (2 Pi) sim /. z → 1 / 20 + I y, {y, -10, 10}]
Out[230]=
-0.632121 + 0. ⅈ
In[231]:=
n2 - n1
Out[231]=
-1. + 0. ⅈ
In[232]:=
Residue[sim, {z, 0}]
Out[232]=
-1
(* B5l2m2 *)
```



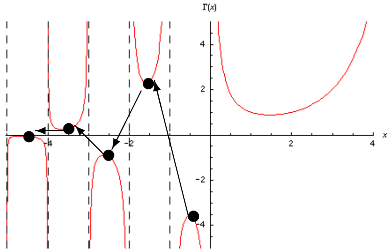
Two basic observations for shifting z follows

$$\int dz_1 \ldots dz_k \ldots I(\ldots, Re[z_k] + n + Im[z_k], \ldots) \qquad I_{orig}$$

$$= \; Residue[\int dz_1 \ldots \not{dz_k} \ldots I]_{Re[z_k]+n} \qquad I_{Res}$$

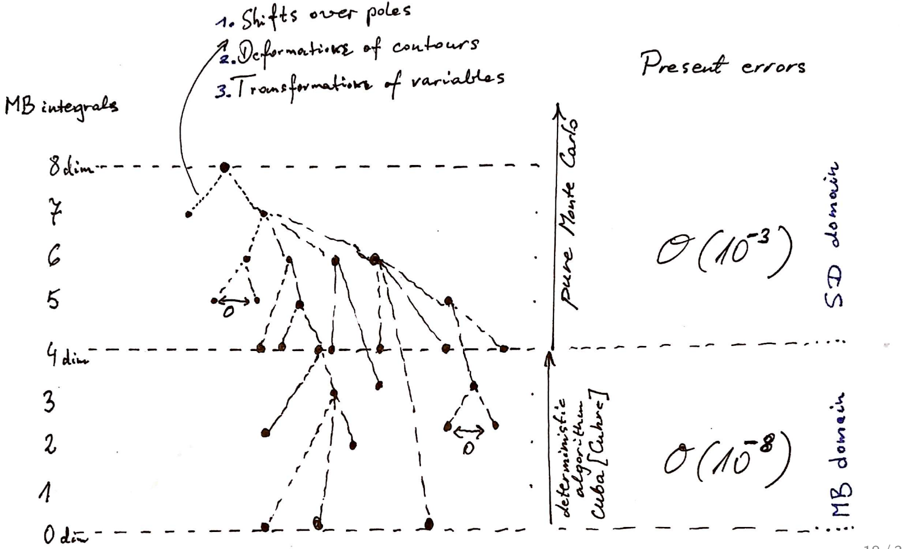
$$+ \; \int dz_1 \ldots dz_k \ldots I(\ldots, Re[z_k] + (n+1) + Im[z_k], \ldots) \qquad I_{new}$$

1. Residues **lower** dimensionality of original MB integrals.
2. Integral after passing a pole (proper shifts) **can be made smaller**.



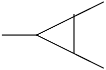
Top-bottom approach to evaluation of multidimensional MB integrals

MBnumerics.m - I. Dubovyk, J. Usovitsch, T. Riemann



BASIC PROBLEMS in Minkowski kinematics

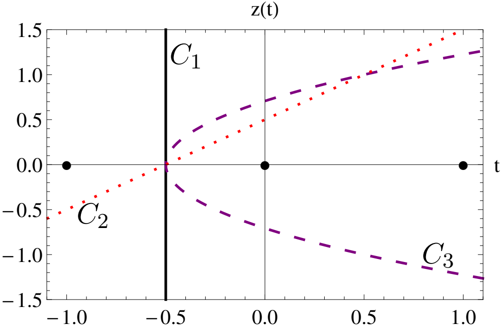
I. Bad oscillatory behavior of integrands;
II. Fragile stability for integrations over products and ratios of Γ functions.



$$\begin{aligned} V(s) &= \frac{e^{\epsilon\gamma_E}}{i\pi^{(4-2\epsilon)/2}} \int \frac{d^{(4-2\epsilon)}k}{[(k+p_1)^2-m^2][k^2][(k-p_2)^2-m^2]} \\ &= \frac{V_{-1}(s)}{\epsilon} + V_0(s) + \cdots , \end{aligned}$$

$$\begin{aligned} V_{-1}(s)|_{m=1} &= -\frac{1}{2s} \int\limits_{-\frac{1}{2}-i\infty}^{-\frac{1}{2}+i\infty} \frac{dz}{2\pi i} \underbrace{(-s)^{-z}}_{\textbf{Problem I}} \overbrace{\frac{\Gamma^3(-z)\Gamma(1+z)}{\Gamma(-2z)}}^{\textbf{Problem II}} \\ &= \frac{1}{2}\sum_{n=0}^{\infty} \frac{s^n}{\binom{2n}{n}(2n+1)} = \frac{2\arcsin(\sqrt{s}/2)}{\sqrt{4-s}\sqrt{s}}, \end{aligned}$$

Contour deformations



$$z(t) = x_0 + it \quad : \quad V_{-1}^{C_1}(s) = \int_{-\infty}^{+\infty} (i)\ dt\ J[z(t)];$$

$$z(t) = x_0 + \theta t + it \quad : \quad V_{-1}^{C_2}(s) = \int_{-\infty}^{+\infty} (\theta + i)\ dt\ J[z(t)]$$

$$z(t) = x_0 + at^2 + it \quad : \quad V_{-1}^{C_3}(s) = \int_{-\infty}^{+\infty} (2at + i)\ dt\ J[z(t)];.$$

$s = 2,\ z(t) = \Re[-1/2] + i\,y, \quad y \in (-a, +a)$

$$V_{-1}(2)|_{\text{analyt.}} = \mathbf{0.785398163397448309 62} = \frac{\pi}{\mathbf{4}}$$

$$V_{-1}(2)|^{MB.m}_{\text{Pantis}} = 0.7925 - \cancel{0.0225\, i}$$

$$V_{-1}(2)|_{C_1,\, a=15} = 0.7548660085063523 - \cancel{0.2299852588 20015\, i}$$

$$V_{-1}(2)|_{C_1,\, a=10^2} = 0.7347931308885253 7844 + \cancel{0.0749014236029376 76597\, i}$$

$$V_{-1}(2)|_{C_1,\, a=10^3} = 0.8471818507353107 6915 - \cancel{0.0948657606493549 77853\, i}$$

$$V_{-1}(2)|_{C_1,\, a=10^4} = 4.4574554985139977 188 + \cancel{4.5139812364645122 275\, i}$$

✓ $$V_{-1}(2)|_{\mathbf{C_2}} = \mathbf{0.785398163}3859819 - 5.42014057525186 4 \cdot 10^{-15 \checkmark}\; i$$

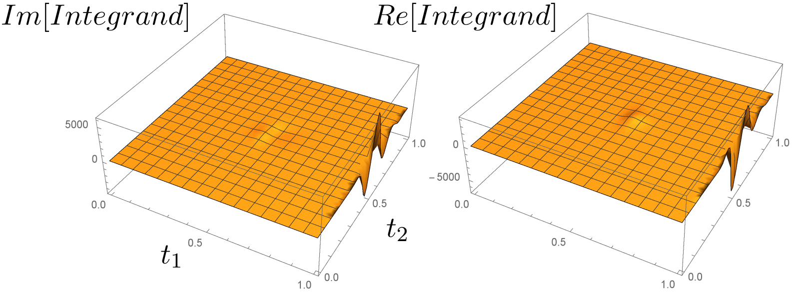
✓ $$V_{-1}(2)|_{\mathbf{C_3}} = \mathbf{0.785398163}2958756 + 2.43555176027143 7 \cdot 10^{-15 \checkmark}\; i$$

Transformations of integration variables (Mappings)

$$\int\limits_{-\frac{1}{3}-i\infty}^{-\frac{1}{3}+i\infty} dz_1 \int\limits_{-\frac{2}{3}-i\infty}^{-\frac{2}{3}+i\infty} dz_2 \left(\frac{-s}{M_Z^2}\right)^{-z_1} \frac{\Gamma[-z_1]^3\Gamma[1+z_1]\Gamma[z_1-z_2]\Gamma[-z_2]^3\Gamma[1+z_2]\Gamma[1-z_1+z_2]}{s\ \ \Gamma[1-z_1]^2\Gamma[-z_1-z_2]\Gamma[1+z_1-z_2]}$$

Logarithmic (in MB.m, M. Czakon, CPC 2006):

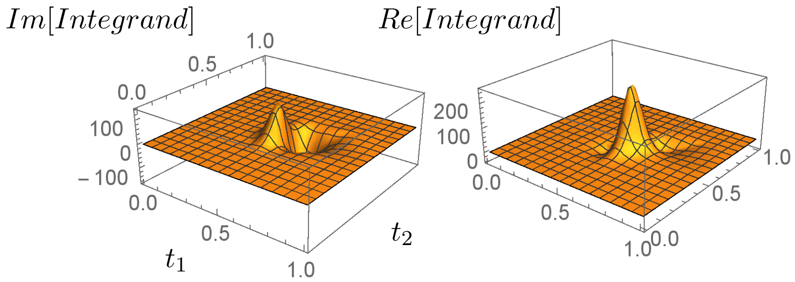
$$z_k \quad = \quad x_k + i\ln\left(\frac{t_k}{1-t_k}\right), \quad t_k \in (0,1), \quad \text{the Jacobians}: \quad J_k(t_k) = \frac{1}{t_k(1-t_k)}.$$



Transformations of variables (Mappings)

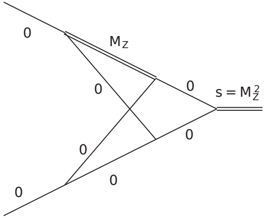
Tangent (in MBnumerics.m, ID, JU, TR, 2016):

$$z_k \quad = \quad x_k + i\, \frac{1}{\tan(-\pi t_k)}, \quad t_k \in (0,1), \quad \text{the Jacobians}: \quad J_k = \frac{\pi}{\sin^2\left[(\pi t_k)\right]}$$



In addition, $\Gamma \to e^{\ln \Gamma}$ improves numerical stability considerable, either.

MB vs SD



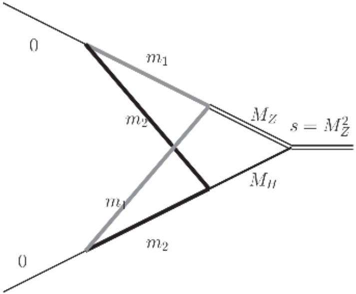
Euclidean results (constant part):

$$\begin{aligned} \text{Analytical}: &\quad -0.4966198306057021 \\ \text{MB(Vegas)}: &\quad -0.4969417442183914 \\ \text{MB(Cuhre)}: &\quad -0.4966198313219404 \\ \text{FIESTA}: &\quad -0.4966184488196595 \\ \text{SecDec}: &\quad -0.4966192150541896 \end{aligned}$$

Minkowskian results (constant part):

$$\begin{aligned} \text{Analytical}: &\quad -0.778599608979684 - 4.123512593396311 \cdot i \\ \text{MBnumerics}: &\quad -0.778599608324769 - 4.123512600516016 \cdot i \\ \text{MB(Vegas)}: &\quad \text{big error} \\ \text{MB(Cuhre)}: &\quad \text{NaN} \\ \text{FIESTA}: &\quad \text{big error} \\ \text{SecDec}: &\quad \text{big error} \end{aligned}$$

Samples of Feynman integral topologies for the $Z\bar{b}b$ vertex



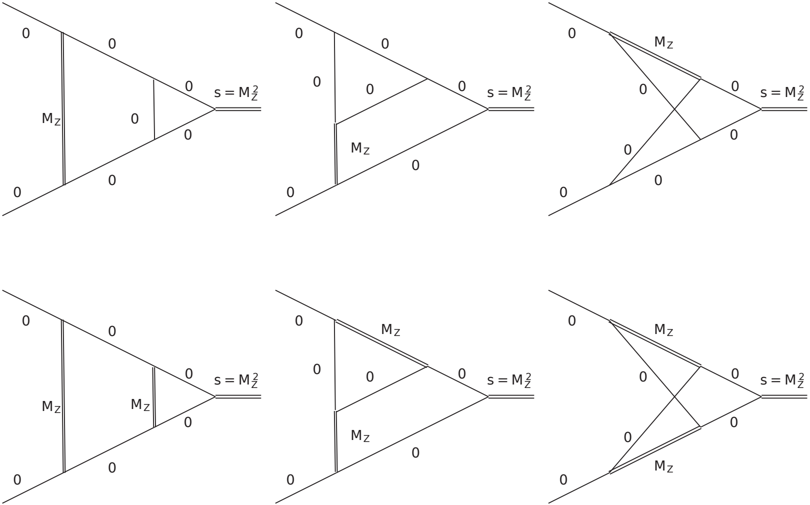
8-dim MB integral (difficult case)

$$m_1 = M_t, m_2 = M_W$$

The integrals contain up to three dimensionless parameters

$$\left\{ \frac{M_H^2}{M_Z^2}, \frac{M_W^2}{M_Z^2}, \frac{m_t^2}{M_Z^2}, \frac{(M_Z + i\varepsilon)^2}{M_Z^2} \right\}$$

The most difficult cases (for SD)



Important

MB and SD methods are very much complementary!

- MB works well for hard threshold, on-shell cases, not many internal masses, SD is powerful for integrals with internal masses.
- see e.g.: J. Gluza, K. Kajda, T. Riemann and V. Yundin
  "Numerical Evaluation of Tensor Feynman Integrals in Euclidean Kinematics"
  Eur. Phys. J. C **71** (2011) 1516; [arXiv:1010.1667 [hep-ph]]

$e^+e^- \to (\gamma, Z) \to l\bar{l}, b\bar{b}, \ldots$

- The standard model prediction for the effective weak mixing angle can be written as

$$\sin^2\theta^{\rm b}_{\rm eff} = \left(1 - \frac{M_W^2}{M_Z^2}\right)(1 + \Delta\kappa_{\rm b})$$

- The bosonic electroweak two-loop corrections amount to

$$\Delta\kappa_{\rm b}^{(\alpha^2,{\rm b}os)} = -0.9855 \times 10^{-4}$$

- 1986 - 1-loop leptonic and $b\bar{b}$ EW corrections (Akhundov, Bardin, Riemann)
- 2006 - 2-loop leptonic EW corrections (Awramik, Czakon, Freitas)
- 2008 - 2-loop $b\bar{b}$ EW corrections with fermionic sub-loops (Awramik, Czakon, Freitas, Kniehl)
- 2016 - Completion: 2-loop $b\bar{b}$ bosonic EW corrections - DFGRU

Collection of radiative corrections: full stabilization at 10^{-4}!

Order	Value $[10^{-4}]$	Order	Value $[10^{-4}]$
α	468.945	$\alpha_t^2\alpha_s$	1.362
$\alpha\alpha_s$	-42.655	α_t^3	0.123
$\alpha_t\alpha_s^2$	-7.074	$\alpha^2_{\rm ferm}$	**3.866**
$\alpha_t\alpha_s^3$	-1.196	$\alpha^2_{\rm bos}$	-0.986

Table: *Comparison of different orders of radiative corrections to* $\Delta\kappa_{\rm b}$*.*
Input Parameters: M_Z, Γ_Z, M_W, Γ_W, M_H, m_t, α_s *and* $\Delta\alpha$

1-loop contributions	Akhundov:1985
fermionic EW 2-loop corrections	Awramik:2008
$\mathcal{O}(\alpha\alpha_{\rm s})$ QCD corrections	Djouadi:1987,Djouadi:1987,Kniehl:1989,Kniehl: 1991, Fleischer:1992,Buchalla:1992,Czarnecki:1996
partial higher-order corrections of orders $\mathcal{O}(\alpha_{\rm t}\alpha_{\rm s}^2)$	Avdeev:1994,Chetyrkin:1995
$\mathcal{O}(\alpha_{\rm t}\alpha_{\rm s}^3)$	Schroder:2005,Chetyrkin:2006,Boughezal:2006
$\mathcal{O}(\alpha^2\alpha_{\rm t})$ and $\mathcal{O}(\alpha_{\rm t}^3)$	vanderBij:2000,Faisst:2003

Bottom-Top approach: JG, Tomasz Jeliński, David Kosower, arXiv:1609.09111

In a nutshell:

- This is a stationary phase method leading to optimal steepest decent contours.
- Contours found using Lefschetz thimbles (exact contours) or Pade approximations.

Lefschetz thimbles (LT) is a fascinating subject, crossing many issues (far way to real multidimensional MB applications):

- Behaviour of LT in presence of poles, singularities and branch cuts, behaviour in the complex infinity, Stokes phenomenon, relation to relative homology of a punctured Riemann sphere etc.
- Other applications of LT: analytical continuation of 3d Chern-Simons theory, QCD with chemical potential, resurgence theory, counting master integrals, repulsive Hubbard model,...

Some basic technicalities of Lefschetz thimbles

- Deform the original integration contour $\mathcal{C}_0$ to a Lefschetz thimble $\mathcal{J}(z_*)$, $f = \mathrm{Re}\, f + i\,\mathrm{Im}\, f$

$$\int_{\mathcal{C}_0} dz\, e^{-f} = \underbrace{e^{-i\,\mathrm{Im}\, f|_{\mathcal{J}(z_*)}}}_{\textbf{Overal factor}} \underbrace{\int_{\mathcal{J}(z_*)} dz\, e^{-\mathrm{Re}\, f}}_{\textbf{Damping factor}} + 2\pi i \underbrace{\sum_{\mathcal{C}_0 \to \mathcal{J}(z_*)} \mathrm{Res}\, e^{-f}}_{\textbf{Remnants}}$$

- Definition: Lefschetz thimble $\mathcal{J}(z_*)$ is a union of curves $t \to z(t) \in \mathbb{C}^n$ which satisfy

$$\underbrace{\frac{d\mathbf{z^i(t)}}{dt}}_{\textbf{Contour}} \overset{!}{=} -\underbrace{\left(\frac{\partial \mathbf{f(z)}}{\partial z^i}\right)^*}_{\textbf{function}}, \quad z(+\infty) = z_*,$$

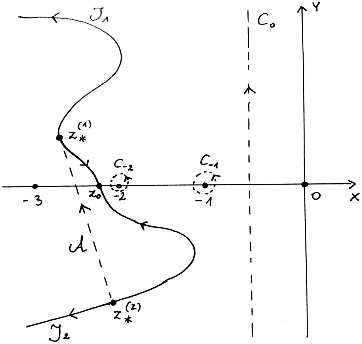
where z_* is a saddle point of a meromorphic function f.

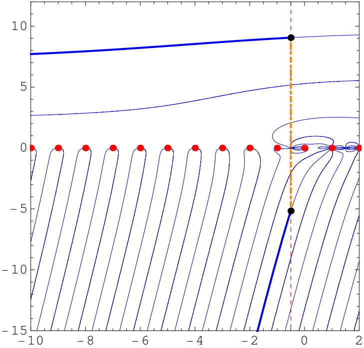
- Properties of $\mathcal{J}(z_*)$: $\mathrm{Im}\, f$ = const. while $\mathrm{Re}\, f$ is monotonically decreasing when $t \to +\infty$ and goes to $+\infty$ when $t \to -\infty$.
- $\mathrm{Im}\, f$ generates Hamiltonian flow on $\mathbb{R}^{2n}$, e.g. for $n = 1$:

$$\frac{dx(t)}{dt} = \frac{\partial \mathrm{Im}\, f}{\partial y}, \quad \frac{dy(t)}{dt} = -\frac{\partial \mathrm{Im}\, f}{\partial x}.$$

Bottom-Top approach: JG, Tomasz Jeliński, David Kosower, arXiv:1609.09111

Exact Lefschetz contour





Shortcut (simplifies integration)

<u>Basic literature:</u>

D. Harlow, J. Maltz, and E. Witten, "Analytic Continuation of Liouville Theory",

L. Nicolaescu, "An Invitation to Morse Theory"

Y. Tanizaki and T. Koike, "Real-time Feynman path integral with Picard-Lefschetz theory and its applications to quantum tunneling"

Further potential applications

- Evaluation of other pseudoobservables, Γ_{Zbb}, $\Gamma_{Z_{\rm tot}}$,... $\longrightarrow$ FCC/ILC,...
- Further development of MB tools;
- Further applications: e.g. including box diagrams (cross sections).

Ayres Freitas, Yi-Cheng Huang, JHEP, 2010

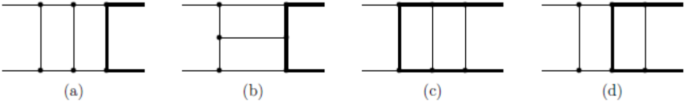


Figure 6. Some scalar master integrals relevant for two-loop QCD corrections to $pp \to t\bar{t}$. Thick lines indicate massive top-quark propagators, while thin lines are massless, and external legs are constrained to be on-shell.

(s, t, m_t^2)	Result from MB representation $\times 10^{15}$ (with integration error)
$(-100^2, -100^2, 175^2)$	$-2.95397\varepsilon^{-2} + [50.15(1) + 0.00(1)i]\varepsilon^{-1} - [515.1(4) - 0.0(4)i]$
$(400^2, -100^2, 175^2)$	$[17.9514 - 13.6206i]\varepsilon^{-2} + [4720.1(5) - 668.5(4)i]\varepsilon^{-1}$ $+[3651(13) - 1481(13)i]$

Table 4. Numerical results for the box diagram in figure 6 (c), obtained with the MB representation method. Monte-Carlo integration errors are given in parentheses. These numbers are based on $m_t = 175$ for the top mass and $\mu = 1$ for the regularization scale.

Conclusions and **C**ontour+**M**ellin+**B**arnes+2017 Epiphany message to You

- There is a progress in direct calculation of multiloop integrals (Feynman diagrams) in the last few years in physical, Minkowski kinematical regime using sector decomposition method
  $\longrightarrow$ SecDec, Fiesta
- In 2016, Mellin-Barnes methods developed for the same goals
  $\longrightarrow$ MBnumerics
- Complementarity of SD and MB methods in many difficult places.

Using numerical methods we are approaching automation in calculation of Feynman integrals beyond the NLO level directly in physical kinematics.