

Report from the "EW precision calculations" miniWorkshop

Indico: Mini-workshop on multiloop/multiscale methods and techniques in the context of precise Z-boson studies

Janusz Gluza

Thanks for help to:

Alain Blondel, Patrick Janot (co-organizers)

Tord Riemann, Staszek Jadach (guidelines)

2nd FCC Physics Workshop

CERN, 15 January 2018

Partly supported from FCC design study and Narodowe Centrum Nauki 2017/25/B/ST2/01987.

CERN, 12-13 January 2018

Indico: Mini-workshop on multiloop/multiscale methods and techniques in the context of precise Z-boson studies

Very good feedback: 32 registered participants, 17 talks



Saturday, 4 pm

21+2 people

Talks, Friday

- Alain Blondel: **Motivation: Experimental capabilities and requirements**
- Janusz Gluza: **Theory - status**
- Tord Riemann: **The Z boson line shape - from 1.5 loops at LEP to 2.5 loops in future**
- Staszek Jadach: **How to calculate QED higher orders in a form useful for MC event generators with soft photon and/or collinear resummation?**
- Vladimir Smirnov: **A mini review of methods of evaluating Feynman integrals. Solving differential equations for Feynman integrals by expansions near singular points**
- Evgen Dubovyk: **MB-suite 1: AMBRE news: non-planar 3-loop vertices**
- Johann Usovitsch: **MB-suite 2: MBnumerics news**
- Sophia Carola Borowka: **pySecDec for phenomenological predictions**
- Roberto Pittau: **Direct calculation of multiloop integrals in $d=4$**

Talks, Saturday

- [Johannes Henn](#): **Bootstrapping pentagon functions**
- [Stefan Weinzierl](#): **Exploring the function space of Feynman integrals**
- [Harald Ita](#): **Expectations from Current Multi-loop Computations with the Unitarity Method**
- [Mario Prausa](#): **Mellin-Barnes meets Method of Brackets**
- [Wojciech Flieger](#): **Steepest descent, homology, Lefschetz thimbles and numerical multidimensional MB integrals**
- [Konstantinos Papadopoulos](#): **About cuts and differential eqs approach**
- [Oleksandr Gituliar](#): **Fuchsian and differential equations for multi-scale master integrals**
- [Peter Marquard](#): **Top quark pair production at threshold and four-loop on-shell renormalization**
- [Rutger Boels](#): **Four loop form factor in $N=4$**

Freeman Dyson:

"New directions in science are launched by new tools much more often than by new concepts."

*The effect of a **concept-driven** revolution is to explain old things in new ways.*

*The effect of a **tool-driven revolution** is to discover new things that have to be explained"*

A. Blondel: FCC-ee experimental demands to theory

observable	Physics	Present precision		FCC-ee stat Syst Precision	FCC-ee key	Challenge
M_Z MeV/c ²	Input	91187.5 ± 2.1	Z Line shape scan	0.005 MeV $< \pm 0.1$ MeV	E_cal	QED corrections
Γ_Z MeV/c ²	$\Delta\rho$ (T) (no $\Delta\alpha$!)	2495.2 ± 2.3	Z Line shape scan	0.008 MeV $< \pm 0.1$ MeV	E_cal	QED corrections
$R_l \equiv \frac{\Gamma_h}{\Gamma_l}$	α_s, δ_b	20.767 (25)	Z Peak	0.0001 (2-20)	Statistics	QED corrections
N_ν	Unitarity of PMNS, sterile ν 's	2.984 ± 0.008	Z Peak Z $+\gamma$ (161 GeV)	0.00008 (40) 0.001	->lumi meast Statistics	QED corrections to Bhabha scat.
R_b	δ_b	0.21629 (66)	Z Peak	0.000003 (20-60)	Statistics, small IP	Hem. corr, gluon split. m_b
A_{LR}	$\Delta\rho, \epsilon_3, \Delta\alpha$ (T, S)	$\sin^2\theta_w^{\text{eff}}$ 0.23098(26)	Z peak, Long. polarized	$\sin^2\theta_w^{\text{eff}}$ ± 0.000006	4 bunch scheme	Design experiment
A_{FB}^{lept}	$\Delta\rho, \epsilon_3, \Delta\alpha$ (T, S)	$\sin^2\theta_w^{\text{eff}}$ 0.23099(53)		$\sin^2\theta_w^{\text{eff}}$ ± 0.000006	E_cal & Statistics	
M_W MeV/c ²	$\Delta\rho, \epsilon_3, \epsilon_2, \Delta\alpha$ (T, S, U)	80385 ± 15	Threshold (161 GeV)	0.3 MeV < 0.5 MeV	E_cal & Statistics	QED corections
m_{top} MeV/c ²	Input	173200 ± 900	Threshold scan	~ 10 MeV	E_cal & Statistics	Theory limit at 50 MeV? ¹⁰

13/01/2018

Main issue

A. Blondel: A BIG QUESTION

Can theory in 2040 (\simeq data taking)
comply with the level of anticipated
experimental accuracy?

To answer, in this talk I will discuss:

- Case of Γ_Z intrinsic accuracy (preliminary results) vs. exp. demand **[0.1 MeV]**;

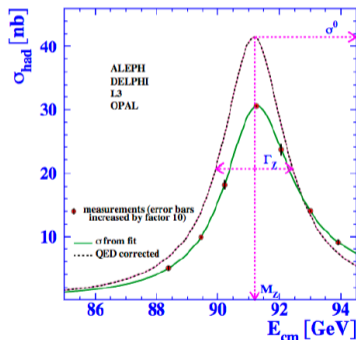
A. Blondel: FCC-ee experimental demands to theory



Example: Precision needed for $e^+e^- \rightarrow \mu^+\mu^-$

- Cross section : Z mass and width

S. Jadach et al.



(Shown here for
 $e^+e^- \rightarrow \text{hadrons}$)

*similar improvement needed for
 $e^+e^- \rightarrow WW$ pair threshold for
 m_W measurement*

- ◆ -30% QED corrections (ISR)
 - Current (LEP) precision translates to an uncertainty of 2×10^{-4} on σ
 - ➔ Improvement by a factor ~5 needed

Alain Blondel FCC-ee workshop on precision calculations



Logical structure of the workshop (1): Basic issues

What we need:

- Calculations at $\sqrt{s} \stackrel{!}{=} M_Z$ and around
→ see [Tord's talk](#) for line shape studies;
- Calculations for a clean setup of EWPOs at fixed order of virtual corrections
→ see [Staszek's talk](#) on implementing higher order QED effects to MC, and resummations;

σ_0 seen from experiments (needs knowing QED)

EWPOs (electroweak pseudo-observables):

$$\sigma_{peak}^{real} \longrightarrow \left\{ \begin{array}{l} \sigma_0 \equiv \sigma_{peak}^{eff.,Born} \\ M_Z, \Gamma_Z, \Gamma_{partial} \\ A_{FB,peak}^{eff.,Born}, A_{LR,peak}^{eff.,Born} \\ R_b, R_\ell \end{array} \right.$$

- Not got for free! **Unfolding of QED issues raised in talks by Tord and Staszek.** Improvements needed for basic LEP programs: KKMC, ZFITTER,...

Don't reinvent the wheel, but for accuracy needed...

- Unfolding of QED effects not got for free!
- Improvements needed for basic LEP programs: KKMC, ZFITTER,...
(ideally independent programs!)
- Issues raised in talks by Tord and Staszek.



EW SM theory at loops, an example ($\Delta_{ef} \neq 0$)

$$\left\{ \begin{array}{l} \Gamma_Z, \Gamma_{\text{partial}} \\ A_{FB, \text{peak}}^{\text{eff., Born}}, A_{LR, \text{peak}}^{\text{eff., Born}} \\ R_b, R_\ell, \dots \end{array} \right. \longrightarrow \left\{ \begin{array}{l} v_{\ell, \nu, u, d, b}^{\text{eff}} \\ a_{\ell, \nu, u, d, b}^{\text{eff}} \\ \sin^2 \theta_{\text{eff}}^b, \sin^2 \theta_{\text{eff}}^{\text{lept}} \end{array} \right.$$

e.g. : improvements needed for subtle corrections $\Delta_{1,2}$ (e.g. boxes, **2L-boxes**)

$$A_{FB, \text{peak}}^{\text{eff., Born}} = \frac{2\Re \left[\frac{v_e a_e^*}{|a_e|^2} \right] 2\Re \left[\frac{v_f a_f^*}{|a_f|^2} \right]}{\left(1 + \frac{|v_e|^2}{|a_e|^2} \right) \left(1 + \frac{|v_f|^2}{|a_f|^2} \right)} + \Delta_1 - \Delta_2 \simeq \frac{3}{4} A_e A_f,$$

$$\Delta_1 = 2\Re [\Delta_{ef}], \quad \Delta_2 = |\Delta_{ef}|^2 + 2\Re \left[\frac{v_e a_e^*}{|a_e|^2} \frac{v_f a_f^*}{|a_f|^2} \Delta_{ef}^* \right],$$

$$\Delta_{ef} = 16 |Q_e Q_f| s_W^4 (\kappa_{ef} - \kappa_e \kappa_f)$$

From Theory: The way to keep a proper track is the S-matrix approach

$$e^+e^- \longrightarrow (\gamma, Z) \longrightarrow f\bar{f}(\gamma),$$

$$A_0^{e^+e^- \rightarrow f\bar{f}} = \frac{R_Z}{s - M_Z^2 + i\Gamma_Z M_Z} + S_0 + (s - s_Z)S' + \dots,$$

$$\sigma_0 \simeq |A_0|^2 = \frac{sr + (s - M_Z^2)j_{int}^{\gamma-Z}}{|s - s_Z^2|^2} + corr.[background]$$

$$r = (v_e^2 + a_e^2)(v_f^2 + a_f^2) + \dots$$

[Willenbrock, Valencia,1991] [Sirlin,1991] [Stuart,1991] [Riemann, 1991, 1992] [H. Veltman,1994]

[Passera, Sirlin, 1998] [Gambino, Grassi, 2000] [Awramik, Czakon, Freitas, 2006].

Solved issue: 2-loop vertex, present issue: 3-loop vertex, potential issue:
2-loop box **(No program implemented 2nd,3rd issue so far!)**

Lesson from Staszek

Main message of this talk:



- ▶ If you calculate multiloop SM radiative corections including QED photonic part, DO NOT follow Bloch-Nordsieck!!!
- ▶ That is DO NOT ADD REAL soft photons to kill infrared singularities of virtual loops!!!
- ▶ Instead, SUBTRACT VIRTUAL part of the-Sudakov Yennie-Frautschi-Suura type from your virtual loop calculations.
- ▶ Why? To make life easier for you and others, more of that follows...



Logical structure of the workshop (2): EWPOs & Form Factors

$$V_\mu^{Zb\bar{b}} = \gamma_\mu [v_b(s) - a_b(s)\gamma_5] = \dots + \underbrace{\left(\underbrace{\text{fermionic, bosonic}}_{\text{planar, non-planar}} \right)}_{\text{planar, non-planar}} + \dots$$

(\rightarrow this talk, see also talks by Rutger and Peter; Johann and Evgen)

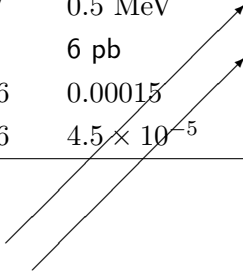
Note approximate factorization of weak couplings

$$A_{F-B} = \frac{\left[\int_0^1 d \cos \theta - \int_{-1}^0 d \cos \theta \right] \frac{d\sigma}{d \cos \theta}}{\sigma_T} \sim \underbrace{\frac{2a_e v_e}{a_e^2 + v_e^2}}_{A_e} \underbrace{\frac{2a_b v_b}{a_b^2 + v_b^2}}_{A_b} + \text{corrections} \leftarrow \text{(Tord)}$$

$$A_b = \frac{2\Re \frac{v_b}{a_b}}{1 + \left(\Re \frac{v_b}{a_b} \right)^2} = \frac{1 - 4|Q_b| \sin^2 \theta_{\text{eff}}^b}{1 - 4|Q_b| \sin^2 \theta_{\text{eff}}^b + 8Q_b^2 (\sin^2 \theta_{\text{eff}}^b)^2}, \quad \sin^2 \theta_{\text{eff}}^b \rightarrow F \left(\Re \frac{v_b}{a_b} \right)$$

Current uncertainties, Ayres: 1604.00406

	Experiment	Theory error	Main source
M_W	80.385 ± 0.015 MeV	4 MeV	$\alpha^3, \alpha^2\alpha_s$
Γ_Z	2495.2 ± 2.3 MeV	0.5 MeV	$\alpha_{\text{bos}}^2, \alpha^3, \alpha^2\alpha_s, \alpha\alpha_s^2$
σ_{had}^0	41540 ± 37 pb	6 pb	$\alpha_{\text{bos}}^2, \alpha^3, \alpha^2\alpha_s$
$R_b \equiv \Gamma_Z^b / \Gamma_Z^{\text{had}}$	0.21629 ± 0.00066	0.00015	$\alpha_{\text{bos}}^2, \alpha^3, \alpha^2\alpha_s$
$\sin^2 \theta_{\text{eff}}^\ell$	0.23153 ± 0.00016	4.5×10^{-5}	$\alpha^3, \alpha^2\alpha_s$



This talk: very preliminary results will be shown and discussed

Future projections, Ayres: 1604.00406

	Measurement error			Intrinsic theory	
	ILC	CEPC	FCC-ee	Current	Future [†]
M_W [MeV]	3–4	3	1	4	1
Γ_Z [MeV]	0.8	0.5	0.1	0.5	0.2
R_b [10^{-5}]	14	17	6	15	7
$\sin^2 \theta_{\text{eff}}^\ell$	1	2.3	0.6	4.5	1.5

Table: Projected experimental and theoretical uncertainties for some electroweak precision pseudo-observables.

[†] Based on estimations for: $\mathcal{O}(\alpha_{bos}^2)$, $\mathcal{O}(\alpha\alpha_s^2)$, $\mathcal{O}(N_f\alpha^2\alpha_s)$, $\mathcal{O}(N_f^2\alpha^3)$

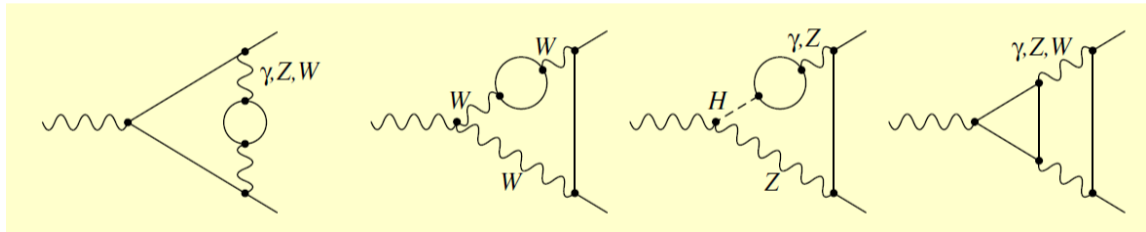
Our main goal ahead:

Present theoretical accuracy is not enough

... we need calculate this kind of corrections, very usefull workshop to show where we are...

Vocabulary

Known corrections ($\Delta\rho$, $\sin^2 \theta_{\text{eff}}^f$, g_V , g_A) comes from fermionic part (fermions loops)



and rest constitute so-called bosonic corrections.

Published results on EWPOs in the SM @NNLO

Complete corrections $\Delta r, \sin^2 \theta_{\text{eff}}^l$:

Freitas, Hollik, Walter, Weiglein: '00

Awramik, Czakon: '02, Onishchenko, Veretin: '02

Awramik, Czakon, Freitas, Weiglein: '04

Awramik, Czakon, Freitas: '06

Hollik, Meier, Uccirati: '05, '07

Degrassi, Gambino, Giardino: '14

Fermionic corrections $\sin^2 \theta_{\text{eff}}^b, a_f, v_f$:

Awramik, Czakon, Freitas, Kniehl: '09

Czarnecki, Kühn: '96

Harlander, Seidensticker, Steinhauser: '98

Freitas: '13, '14

Bosonic corrections $\sin^2 \theta_{\text{eff}}^b$:

Dubovyk, Freitas, JG, Riemann, Usovitsch '16

This talk: Bosonic corrections a_f, v_f :

Dubovyk, Freitas, JG, Riemann, Usovitsch '18

What we need: error estimations, Ayres: 1604.00406

- Theory error estimate is not well defined, ideally $\Delta_{\text{th}} \ll \Delta_{\text{exp}}$
- Common methods:
 - Count prefactors (α, N_c, N_f, \dots)
 - Extrapolation of perturbative series
 - Renormalization scale dependence
 - Renormalization scheme dependence
- Also parametric error from external inputs ($m_t, m_b, \alpha_s, \Delta\alpha_{\text{had}}, \dots$)

An example: Intrinsic theory error estimation for Γ_Z , Ayres: 1604.00406

① Geometric series

$$\delta_1 : \mathcal{O}(\alpha^3) - \mathcal{O}(\alpha_t^3) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha^2) \sim 0.26 \text{ MeV}$$

$$\delta_2 : \mathcal{O}(\alpha^2 \alpha_s) - \mathcal{O}(\alpha_t^2 \alpha_s) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha \alpha_s) \sim 0.3 \text{ MeV}$$

$$\delta_3 : \mathcal{O}(\alpha \alpha_s^2) - \mathcal{O}(\alpha_t \alpha_s^2) \sim \frac{\mathcal{O}(\alpha \alpha_s) - \mathcal{O}(\alpha_t \alpha_s)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha \alpha_s) \sim 0.23 \text{ MeV}$$

$$\delta_4 : \mathcal{O}(\alpha \alpha_s^3) - \mathcal{O}(\alpha_t \alpha_s^3) \sim \frac{\mathcal{O}(\alpha \alpha_s) - \mathcal{O}(\alpha_t \alpha_s)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha \alpha_s^2) \sim 0.035 \text{ MeV}$$

$$\delta_5 : \mathcal{O}(\alpha_{bos}^2) \sim \mathcal{O}(\alpha_{bos})^2 \sim \mathbf{0.1 \text{ MeV}} \text{ [Now we know it!]}$$

$$\text{Total: } \delta\Gamma_Z = \sqrt{\sum_{i=1}^5 \delta_i^2} \sim \mathbf{0.5 \text{ MeV}}$$

New results for completing NNLO

Input parameters:

Parameter	Value	Parameter	Value
M_Z	91.1876 GeV	$m_b^{\overline{\text{MS}}}$	4.20 GeV
Γ_Z	2.4952 GeV	$m_c^{\overline{\text{MS}}}$	1.275 GeV
M_W	80.385 GeV	m_τ	1.777 GeV
Γ_W	2.085 GeV	$\Delta\alpha$	0.05900
M_H	125.1 GeV	$\alpha_s(M_Z)$	0.1184
m_t	173.2 GeV	G_μ	$1.16638 \times 10^{-5} \text{ GeV}^{-2}$

The cherry on the 2-loops EWPOs cake: results for $\mathcal{O}(\alpha_{\text{bos}}^2)$

[preliminary]*

Γ_i [MeV]	$\Gamma_e, \Gamma_\mu, \Gamma_\tau$	$\Gamma_{\nu_e}, \Gamma_{\nu_\mu}, \Gamma_{\nu_\tau}$	Γ_d, Γ_s	Γ_u, Γ_c	Γ_b	Γ_Z
$\mathcal{O}(\alpha)$	2.273	6.174	9.717	5.799	3.857	60.22
$\mathcal{O}(\alpha\alpha_s)$	0.288	0.458	1.276	1.156	2.006	9.11
$\mathcal{O}(N_f^2\alpha^2)$	0.244	0.416	0.698	0.528	0.694	5.13
$\mathcal{O}(N_f\alpha^2)$	0.120	0.185	0.493	0.494	0.144	3.04
$\mathcal{O}(\alpha_{\text{bos}}^2)$	0.017	0.019	0.058	0.057	0.167	0.505
$\mathcal{O}(\alpha_t\alpha_s^2, \alpha_t\alpha_s^3, \alpha_t^2\alpha_s, \alpha_t^3)$	0.038	0.059	0.191	0.170	0.190	1.20

- ① Fun fact of the day: so far all contributions positive!
- ② 2016, estimation, bosonic NNLO $\sim 0 \pm 0.1$ MeV
2018, exact result: 0.505 MeV

* I. Dubovyk, A. Freitas, JG, T. Riemann, J. Usovitsch, see talk by JG at miniWorkshop

Having this knowledge: **genuine** 3-loop vertex calculations are obligatory!

① Geometric series

$$\mathcal{O}(\alpha^3) - \mathcal{O}(\alpha_t^3) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha^2) \sim \underline{0.26 \text{ MeV}}$$

$$\mathcal{O}(\alpha^2 \alpha_s) - \mathcal{O}(\alpha_t^2 \alpha_s) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha \alpha_s) \sim \underline{0.3 \text{ MeV}}$$

$$\mathcal{O}(\alpha \alpha_s^2) - \mathcal{O}(\alpha_t \alpha_s^2) \sim \frac{\mathcal{O}(\alpha \alpha_s) - \mathcal{O}(\alpha_t \alpha_s)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha \alpha_s) \sim \underline{0.23 \text{ MeV}}$$

$$\mathcal{O}(\alpha \alpha_s^3) - \mathcal{O}(\alpha_t \alpha_s^3) \sim \frac{\mathcal{O}(\alpha \alpha_s) - \mathcal{O}(\alpha_t \alpha_s)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha \alpha_s^2) \sim \underline{0.035 \text{ MeV}}$$

$$\mathcal{O}(\alpha_{bos}^2) \sim \mathcal{O}(\alpha_{bos})^2 \sim \underline{0.1 \text{ MeV}} \rightarrow \mathbf{[0.51 \text{ MeV}]}$$

① FCC-ee ^{exper. error} $(\Gamma_Z) \sim 0.1 \text{ MeV}$

② FCC-ee ^{theor. error} $(\Gamma_Z) < \text{FCC-ee}^{\text{exper. error}}(\Gamma_Z) ???$

Answering Alain Blondel's Big Question

- ① Calculating N^3LO with 10% accuracy (two digits), we can replace 2016 intrinsic error estimation $\delta\Gamma_Z = \sqrt{\sum_{i=1}^5 \delta_i^2} \sim 0.5$ MeV by

$$\delta\Gamma_Z = \sqrt{\sum_{i=1}^5 (\delta_i/10)^2} \sim 0.05 \text{ MeV.}$$

- ① The requirement of FCC-ee^{exper. error}(Γ_Z) ~ 0.1 MeV can be met and the condition

$$\delta[\text{FCCee}^{\text{theor.}}(\Gamma_Z)] \sim 0.05 \text{ MeV} < \delta[\text{FCCee}^{\text{exper.}}(\Gamma_Z)] \sim 0.1 \text{ MeV}$$

will be fulfilled.

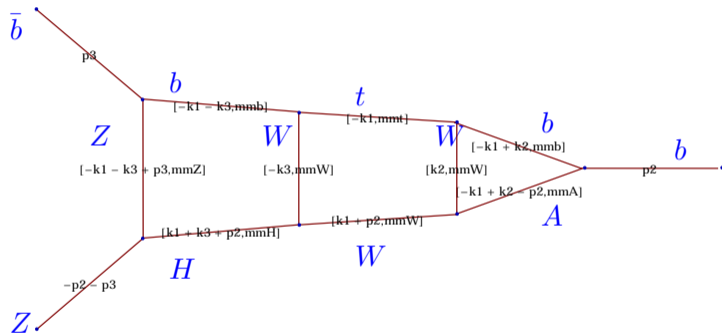
Basic bookkeeping

$Z \rightarrow b\bar{b}$			
Number of topologies	1 loop	2 loops	3 loops
		1	$14 \xrightarrow{(A)}$ $7 \xrightarrow{(B)}$ 5
Number of diagrams	15	$2383 \xrightarrow{(A,B)}$ 1114	$490387 \xrightarrow{(A,B)}$ 120187
Fermionic loops	0	150	17580
Bosonic loops	15	964	102607
Planar diagrams	1T/15D	4T/981D	35T/84059D
Non-planar diagrams	0	1T/133D	15T/36128D

Table: Some statistical overview for $Z \rightarrow b\bar{b}$ multiloop studies. At 3 loops there are in total almost half a million of diagrams present. After basic refinements (A) and (B) about 10^5 genuine 3-loop vertex diagrams remain. In (A) tadpoles and products of lower loops are excluded, in (B) symmetries of topologies are taken into account.

A complete zoo of heavy particles m_t, m_W, m_Z, m_H @NNNLO level

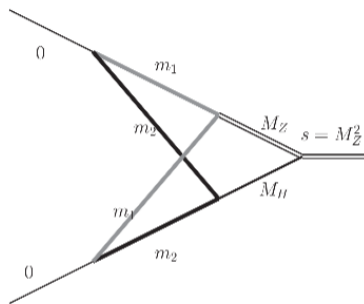
MB: ϵ^0 [8-dim], $1/\epsilon$ [7-dim]; SD: ϵ^0 [8-dim], $1/\epsilon$ [7-dim];



At 2-loops up to three dimensionless parameters (all 4 at 3-loops):

$$\left\{ \frac{M_H^2}{M_Z^2}, \frac{M_W^2}{M_Z^2}, \frac{m_t^2}{M_Z^2}, \frac{(M_Z + i\epsilon)^2}{M_Z^2} \right\}$$

Logical structure of the workshop (3): Methods and Tools



$$m_1 = M_t, m_2 = M_W$$

- ① (Semi-)Analytical approaches, e.g. DEqs, IBPs, special functions, unitarity, ...
 - Volodya,
 - Johannes, Costas, Oleksandr
 - Stefan, Harald, Mario
- ② Numerical approaches, e.g. **MB, SD**, D=4, ...
 - Ievgen, Johann, Sophia,
 - Roberto,
 - Wojciech

We can do it. To substantiate my claim:

An example of effective weak mixing angle $\sin^2 \theta_{\text{eff}}^b$

- The standard model prediction for the effective weak mixing angle can be written as

$$\sin^2 \theta_{\text{eff}}^b = \left(1 - \frac{M_W^2}{M_Z^2} \right) (1 + \Delta\kappa_b)$$

- The bosonic electroweak two-loop corrections amount to

$$\Delta\kappa_b^{(\alpha^2, \text{bos})} = -0.9855 \times 10^{-4}$$

DFGRU, Phys.Lett. B762 (2016) 184

Collection of radiative corrections: Full stabilization at 10^{-4} ! $\pm 0.001 \xrightarrow{!}$

Order	Value [10^{-4}]	Order	Value [10^{-4}]
α	468.945	$\alpha_t^2 \alpha_s$	1.362
$\alpha \alpha_s$	-42.655	α_t^3	0.123
α_{ferm}^2	3.866	$\alpha_t \alpha_s^2$	-7.074
α_{bos}^2	-0.9855	$\alpha_t \alpha_s^3$	-1.196

Table: Comparison of different orders of radiative corrections to $\Delta \kappa_b$.

Input Parameters: $M_Z, \Gamma_Z, M_W, \Gamma_W, M_H, m_t, \alpha_s$ and $\Delta \alpha$

- one-loop contributions [Akhundov, Bardin, Riemann, 1986] [Beenakker, Hollik, 1988]
- two-loop fermionic contributions [Awramik, Czakon, Freitas, Kniehl, 2009]
- two-loop bosonic contributions [Dubovyk, Freitas, JG, Riemann, Usovitsch, 2016]

Partial higher-order corrections

$\mathcal{O}(\alpha_t \alpha_s^2)$

$\mathcal{O}(\alpha_t \alpha_s^3)$

$\mathcal{O}(\alpha^2 \alpha_t)$ and $\mathcal{O}(\alpha_t^3)$

Avdeev: 1994, Chetyrkin: 1995

Schroder: 2005, Chetyrkin: 2006, Boughezal: 2006

vanderBij: 2000, Faisst: 2003

Mellin-Barnes and Sector Decomposition methods are very much complementary

- MB works well for hard threshold, on-shell cases, not many internal masses (more IR); SD more useful for integrals with many internal masses
 - talks by Evgen, Johann and Sophia;
 - JG, Kajda, Riemann, Yundin, EPJC'11; JG in PoS-LL2016 & DFGRU in PLB'16.

10^{-8} accuracy achieved for **any** self-energy and vertex Feynman integral with one of the methods.

Collective effort needed!



and resources needed!

Conclusions on Z-lineshape and EWPOs for next years - theory

- Strong demand from FCC-ee to the theory on precision;
- We have to guarantee precise chain:
 $\sigma^{real} \rightarrow$ pseudoobservables $\rightarrow >$ 2-loops in SM
- NNLO practically done, we need to go beyond:
 $\mathcal{O}(\alpha\alpha_s^2), \mathcal{O}(N_f\alpha^2\alpha_s), \mathcal{O}(N_f^2\alpha^3)$;
- ① **We know how** to do it;
 ② and **we have appropriate tools**;
- To be on the safe side, we would like to have **at least 2 independent calculations**;
- Still, a lot work is ahead, for success and efficiency, **we need steady progress in numerical and also (semi)analytical approaches** in multiloop calculations

Answering Alain Blondel's Big Question

- ① Calculating N^3LO with 10% accuracy (two digits), we can replace 2016 intrinsic error estimation $\delta\Gamma_Z = \sqrt{\sum_{i=1}^5 \delta_i^2} \sim 0.5$ MeV by

$$\delta\Gamma_Z = \sqrt{\sum_{i=1}^5 (\delta_i/10)^2} \sim 0.05 \text{ MeV.}$$

- ① The requirement of FCC-ee^{exper. error}(Γ_Z) ~ 0.1 MeV can be met and the condition

$$\delta[\text{FCCee}^{\text{theor.}}(\Gamma_Z)] \sim 0.05 \text{ MeV} < \delta[\text{FCCee}^{\text{exper.}}(\Gamma_Z)] \sim 0.1 \text{ MeV}$$

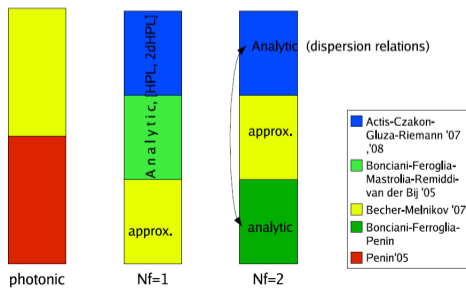
will be fulfilled.

Several reasons to stay optimistic in "microscoping" higher order calculations

- 1 **Steady** progress in numerical calculations, methods and tools;
- 2 Lessons from the past (LEP, LHC,...) - anticipated SM predictions improved considerably - sometimes even several times after experiments took off;
- 3 Often problems can be attacked from different perspectives (it is needed for independent confirmations);

Present situation, virtual NNLO QED

Bhabha scattering, 10 years ago



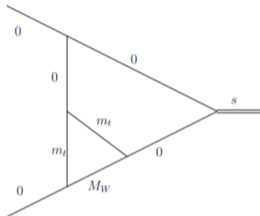
Sector decomposition

FIESTA 3 [A.V.Smirnov, 2014], SecDec 3 [Borowka, et. al., 2015] and pySecDec [Borowka, et. al., 2017]

Mellin-Barnes integral approach

- With AMBRE 2 [Gluza, et. al., 2011] (AMBRE 3 [Dubovyk, et. al., 2015]) we derive Mellin-Barnes representations for planar (non-planar) topologies. One may use PlanarityTest [Bielas, et. al, 2013] for automatic identification.
- Expansion in terms of $\epsilon = (4 - D)/2$ is done with MB [Czakon, 2006], MBresolve [A. Smirnov, V. Smirnov, 2009], barnesroutines (D. Kosower).
- For the numerical treatment of massive Mellin-Barnes integrals with Minkowskian regions, the package MBnumerics is being developed since 2015.

soft7 ϵ^0 : [MB - 3 dim] [SD - 5 dim], ϵ^{-1} : [MB - 2 dim] [SD - 4 dim], ϵ^{-2} : [MB - 1 dim] [SD - 3 dim]



MB	0.060266486557699 9 ϵ^{-2}	
SD - 90 Mio	0.0602664865 5 ϵ^{-2}	
MB	(-0.031512489 03	+0.189332751 42i) ϵ^{-1}
SD - 90 Mio	(-0.03151248 16	+0.18933271 696i) ϵ^{-1}
MB 1	(-0.2282318675 11	-0.0882479456 91i) + $\mathcal{O}(\epsilon)$
MB 2	(-0.2282318675 51	-0.0882479457 39i) + $\mathcal{O}(\epsilon)$
SD - 90 Mio	(-0.228226 53	-0.088245 96i) + $\mathcal{O}(\epsilon)$
SD - 15 Mio	(-0.2281 62	-0.0882 09i) + $\mathcal{O}(\epsilon)$

Intermezzo: 1997 → 2017/2018 → 2038



Twenty years on from Deep Blue vs Kasparov: how a chess match started the big data revolution

May 11, 2017 3.12pm BST

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