Z-boson physics in the context of FCC-ee: theory

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Mini-workshop on multiloop/multiscale methods and techniques
in the context of precise Z-boson studies

CERN, 12 January 2018

Outline

1. Introduction: Z-boson - 50 years of studies
2. Logical structure of the workshop
3. EWPOs, present status and future demands
4. Needs for EWPOs beyond 2-loops
5. Bright perspective: why?
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50 years of the Z-boson theory (1967)

S. Weinberg

"A MODEL OF LEPTONS"

VOLUME 19, NUMBER 21 PHYSICAL REVIEW LETTERS 20 NOVEMBER 1967

and

$$\varphi_1 \equiv (\varphi^0 + \varphi^{0\dagger} - 2\lambda)/\sqrt{2} \qquad \varphi_2 \equiv (\varphi^0 - \varphi^{0\dagger})/i\sqrt{2}. \qquad (5)$$

The condition that φ_1 have zero vacuum expectation value to all orders of perturbation theory tells us that $\lambda^2 \cong M_1^2/2h$, and therefore the field φ_1 has mass M_1 while φ_2 and φ^- have mass zero. But we can easily see that the Goldstone bosons represented by φ_2 and φ^- have no physical coupling. The Lagrangian is gauge invariant, so we can perform a combined isospin and hypercharge gauge transformation which eliminates φ^- and φ_2 everywhere[6] without changing anything else. We will see that G_e is very small, and in any case M_1 might be very large,[7] so the φ_1 couplings will also be disregarded in the following.

The effect of all this is just to replace φ everywhere by its vacuum expectation value

$$\langle\varphi\rangle = \lambda\binom{1}{0}. \qquad (6)$$

The first four terms in $\mathcal{L}$ remain intact, while the rest of the Lagrangian becomes

$$-\tfrac{1}{8}\lambda^2 g^2[(A_\mu^{\ 1})^2 + (A_\mu^{\ 2})^2] - \tfrac{1}{8}\lambda^2(gA_\mu^{\ 3} + g'B_\mu)^2 - \lambda G_e \bar{e}e. \qquad (7)$$

We see immediately that the electron mass is λG_e. The charged spin-1 field is

$$W_\mu \equiv 2^{-1/2}(A_\mu^{\ 1} + iA_\mu^{\ 2}) \qquad (8)$$

and has mass

$$M_W = \tfrac{1}{2}\lambda g. \qquad (9)$$

The neutral spin-1 fields of definite mass are

$$Z_\mu = (g^2 + g'^2)^{-1/2}(gA_\mu^{\ 3} + g'B_\mu), \qquad (10)$$

$$A_\mu = (g^2 + g'^2)^{-1/2}(-g'A_\mu^{\ 3} + gB_\mu). \qquad (11)$$

Their masses are

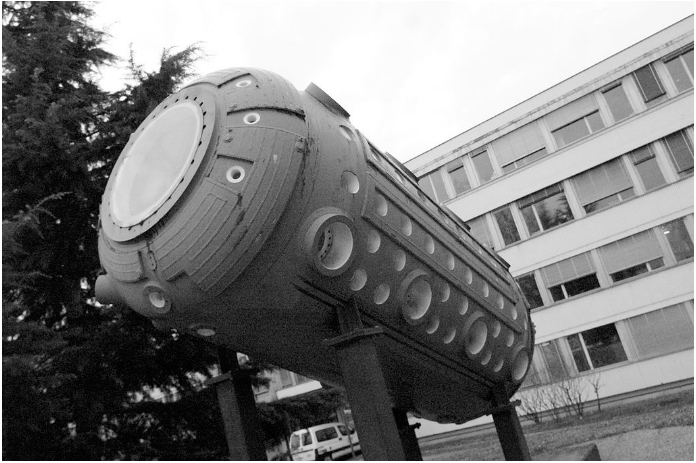
$$M_Z = \tfrac{1}{2}\lambda(g^2 + g'^2)^{1/2}, \qquad (12)$$

$$M_A = 0, \qquad (13)$$

so A_μ is to be identified as the photon field. The interaction between leptons and spin-1 mesons is

$$\frac{ig}{2\sqrt{2}}\bar{e}\gamma^\mu(1+\gamma_5)\nu W_\mu + \text{H.c.} + \frac{igg'}{(g^2+g'^2)^{1/2}}\bar{e}\gamma^\mu e A_\mu + \frac{i(g^2+g'^2)^{1/2}}{4}\left[\left(\frac{3g'^2-g^2}{g'^2+g^2}\right)\bar{e}\gamma^\mu e - \bar{e}\gamma^\mu\gamma_5 e + \bar{\nu}\gamma^\mu(1+\gamma_5)\nu\right]Z_\mu. \qquad (14)$$

And, exactly 45 years of the Z-boson discovery (1973)



just look outside...

Rich physics

Erler, Freitas, PDG'17

Presently:

Very good agreement

theory — experiment

over large number of EWPOs

Table 10.5: Principal Z pole observables and their SM predictions (*cf.* Table 10.4). The first $\bar{s}_\ell^2$ is the effective weak mixing angle extracted from the hadronic charge asymmetry, the second is the combined value from the Tevatron [164–166], and the third from the LHC [170–172]. The values of A_e are (i) from A_{LR} for hadronic final states [159]; (ii) from A_{LR} for leptonic final states and from polarized Bhabba scattering [161]; and (iii) from the angular distribution of the τ polarization at LEP 1. The A_τ values are from SLD and the total τ polarization, respectively.

Quantity	Value	Standard Model	Pull
M_Z [GeV]	91.1876 ± 0.0021	91.1880 ± 0.0020	-0.2
Γ_Z [GeV]	2.4952 ± 0.0023	2.4943 ± 0.0008	0.4
$\Gamma(\text{had})$ [GeV]	1.7444 ± 0.0020	1.7420 ± 0.0008	—
$\Gamma(\text{inv})$ [MeV]	499.0 ± 1.5	501.66 ± 0.05	—
$\Gamma(\ell^+\ell^-)$ [MeV]	83.984 ± 0.086	83.995 ± 0.010	—
σ_{had}[nb]	41.541 ± 0.037	41.484 ± 0.008	1.5
R_e	20.804 ± 0.050	20.734 ± 0.010	1.4
R_μ	20.785 ± 0.033	20.734 ± 0.010	1.6
R_τ	20.764 ± 0.045	20.779 ± 0.010	-0.3
R_b	0.21629 ± 0.00066	0.21579 ± 0.00003	0.8
R_c	0.1721 ± 0.0030	0.17221 ± 0.00003	0.0
$A_{FB}^{(0,e)}$	0.0145 ± 0.0025	0.01622 ± 0.00009	-0.7
$A_{FB}^{(0,\mu)}$	0.0169 ± 0.0013		0.5
$A_{FB}^{(0,\tau)}$	0.0188 ± 0.0017		1.5
$A_{FB}^{(0,b)}$	0.0992 ± 0.0016	0.1031 ± 0.0003	-2.4
$A_{FB}^{(0,c)}$	0.0707 ± 0.0035	0.0736 ± 0.0002	-0.8
$A_{FB}^{(0,s)}$	0.0976 ± 0.0114	0.1032 ± 0.0003	-0.5
$\bar{s}_\ell^2$	0.2324 ± 0.0012	0.23152 ± 0.00005	0.7
	0.23185 ± 0.00035		0.9
	0.23105 ± 0.00087		-0.5
A_e	0.15138 ± 0.00216	0.1470 ± 0.0004	2.0
	0.1544 ± 0.0060		1.2
	0.1498 ± 0.0049		0.6
A_μ	0.142 ± 0.015		-0.3
A_τ	0.136 ± 0.015		-0.7
	0.1439 ± 0.0043		-0.7
A_b	0.923 ± 0.020	0.9347	-0.6
A_c	0.670 ± 0.027	0.6678 ± 0.0002	0.1
A_s	0.895 ± 0.091	0.9356	-0.4

Theoretical playground

EWPOs (electroweak pseudo-observables):

$$\{\Gamma_{Zff},\, \Gamma_{Zbb}\} \quad \longrightarrow \Gamma_{Z_{\mathrm{tot}}}$$
$$R_\ell$$
$$R_{c,b}$$
$$A^f_{\mathrm{FB}},\, A^f_{\mathrm{LR}}$$
$$\sin^2\theta^{\mathrm{b}}_{\mathrm{eff}},\, \sin^2\theta^{lept}_{\mathrm{eff}}$$

In future (Alain's talk) sensitivity of experiments better by a factor 20-100 compared to LEP physics.

Our main goal ahead:

Present theoretical accuracy is not enough,

... and that is why we are here!

Logical structure of the workshop (1): Basic issues

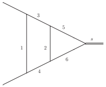
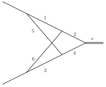
What we need:

- Calculations at $\sqrt{s} \stackrel{!}{=} M_Z$ and around
  $\longrightarrow$ see Tord's talk for line shape studies;
- Calculations for a clean setup of EWPOs at fixed order of virtual corrections
  $\longrightarrow$ see Staszek's talk on implementing higher order QED effects to MC, and resummations;

Logical structure of the workshop (2): EWPOs & Form Factors

$$V_\mu^{Zb\bar{b}} = \gamma_\mu[v_b(s) - a_b(s)\gamma_5] = \cdots +$$

fermionic,bosonic

 $+$ 

planar,non−planar

$+\cdots$

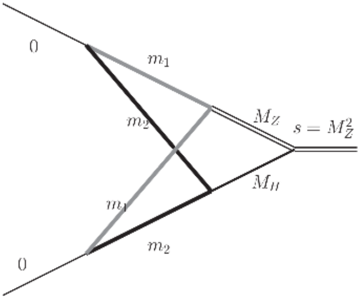
($\longrightarrow$ this talk, see also talks by Rutger and Peter; Johann and Evgen)

Note approximate factorization of weak couplings

$$A_{F-B} = \frac{\left[\int_0^1 d\cos\theta - \int_{-1}^0 d\cos\theta\right]\frac{d\sigma}{d\cos\theta}}{\sigma_T} \sim \overbrace{\frac{2a_e v_e}{a_e^2+v_b^2}}^{A_e}\ \overbrace{\frac{\mathbf{2a_b v_b}}{\mathbf{a_b^2+v_b^2}}}^{A_b} + \text{corrections} \leftarrow \text{(Tord)}$$

$$A_\mathrm{b} = \frac{2\Re e\frac{v_b}{a_b}}{1+\left(\Re e\frac{v_b}{a_b}\right)^2} = \frac{1-4|Q_b|\sin^2\theta_\mathrm{eff}^\mathrm{b}}{1-4|Q_b|\sin^2\theta_\mathrm{eff}^\mathrm{b}+8Q_b^2(\sin^2\theta_\mathrm{eff}^\mathrm{b})^2}, \quad \sin^2\theta_\mathrm{eff}^\mathrm{b} \longrightarrow F\left(\Re e\frac{v_b}{a_b}\right)$$

Logical structure of the workshop (3): Methods and Tools

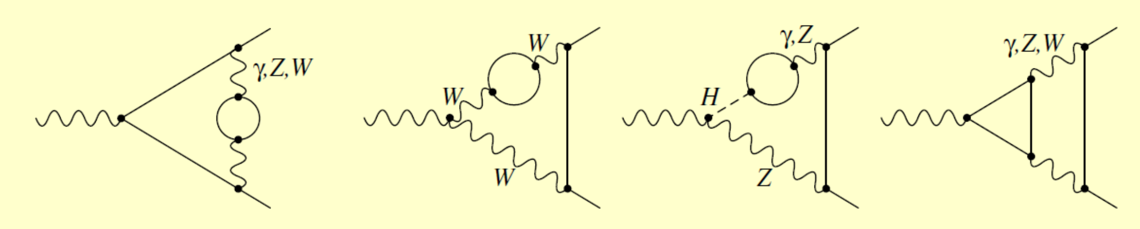


$$m_1 = M_t, m_2 = M_W$$

1. (Semi-)Analytical approaches, e.g. DEqs, IBPs, special functions, unitarity, ...
   $\longrightarrow$ Volodya,
   $\longrightarrow$ Johannes, Costas, Oleksandr
   $\longrightarrow$ Stefan, Harald, Mario
2. Numerical approaches, e.g. **MB, SD**, D=4, ...
   $\longrightarrow$ Ievgen, Johann, Sophia,
   $\longrightarrow$ Roberto,
   $\longrightarrow$ Wojciech

Published results on EWPOs in the SM @NNLO (1)

Known corrections ($\Delta\rho$, $\sin^2\theta^{\rm f}_{\rm eff}, g_V, g_A$):



Fermionic (with fermions loops) and bosonic (rest)

Published results on EWPOs in the SM @NNLO (2)

Complete corrections $\Delta r, \sin^2\theta^l_{\rm eff}$:	Freitas, Hollik, Walter, Weiglein: '00
	Awramik,Czakon: '02,Onishchenko,Veretin: '02
	Awramik,Czakon,Freitas,Weiglein: '04
	Awramik,Czakon,Freitas: '06
	Hollik,Meier,Uccirati: '05,'07
	Degrassi,Gambino,Giardino: '14
Fermionic corrections $\sin^2\theta^b_{\rm eff}, a_f, v_f$:	Awramik,Czakon,Freitas,Kniehl: '09
	Czarnecki,Kühn: '96
	Harlander,Seidensticker,Steinhauser: '98
	Freitas: '13,'14
Bosonic corrections $\sin^2\theta^{\rm b}_{\rm eff}$:	Dubovyk, Freitas, JG, Riemann, Usovitsch '16
This talk: Bosonic corrections a_f, v_f :	Dubovyk, Freitas, JG, Riemann, Usovitsch '18

Current uncertainities, Ayres: 1604.00406

	Experiment	Theory error	Main source
M_{W}	80.385 ± 0.015 MeV	4 MeV	α^3, $\alpha^2\alpha_{\mathrm{s}}$
Γ_{Z}	2495.2 ± 2.3 MeV	0.5 MeV	α^2_{bos}, α^3, $\alpha^2\alpha_{\mathrm{s}}$, $\alpha\alpha^2_{\mathrm{s}}$
σ^0_{had}	41540 ± 37 pb	6 pb	α^2_{bos}, α^3, $\alpha^2\alpha_{\mathrm{s}}$
$R_b \equiv \Gamma^b_{\mathrm{Z}}/\Gamma^{\mathrm{had}}_{\mathrm{Z}}$	0.21629 ± 0.00066	0.00015	α^2_{bos}, α^3, $\alpha^2\alpha_{\mathrm{s}}$
$\sin^2\theta^{\ell}_{\mathrm{eff}}$	0.23153 ± 0.00016	4.5×10^{-5}	α^3, $\alpha^2\alpha_{\mathrm{s}}$

This talk: very preliminary results will be shown and discussed

What we need: error estimations, Ayres: 1604.00406

- Theory error estimate is not well defined, ideally $\Delta_{\mathrm{th}} \ll \Delta_{\mathrm{exp}}$
- Common methods:
  - Count prefactors (α, N_c, N_f, ...)
  - Extrapolation of perturbative series
  - Renormalization scale dependence
  - Renormalization scheme dependence
- Also parametric error from external inputs (m_{t}, m_b, α_{s}, $\Delta\alpha_{\mathrm{had}}$, ...)

An example: Intrinsic theory error estimation for Γ_Z, Ayres: 1604.00406

1. Geometric series

$$\delta_1 : \mathcal{O}(\alpha^3) - \mathcal{O}(\alpha_t^3) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)}\mathcal{O}(\alpha^2) \sim 0.26 \text{ MeV}$$

$$\delta_2 : \mathcal{O}(\alpha^2\alpha_s) - \mathcal{O}(\alpha_t^2\alpha_s) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)}\mathcal{O}(\alpha\alpha_s) \sim 0.3 \text{ MeV}$$

$$\delta_3 : \mathcal{O}(\alpha\alpha_s^2) - \mathcal{O}(\alpha_t\alpha_s^2) \sim \frac{\mathcal{O}(\alpha\alpha_s) - \mathcal{O}(\alpha_t\alpha_s)}{\mathcal{O}(\alpha)}\mathcal{O}(\alpha\alpha_s) \sim 0.23 \text{ MeV}$$

$$\delta_4 : \mathcal{O}(\alpha\alpha_s^3) - \mathcal{O}(\alpha_t\alpha_s^3) \sim \frac{\mathcal{O}(\alpha\alpha_s) - \mathcal{O}(\alpha_t\alpha_s)}{\mathcal{O}(\alpha)}\mathcal{O}(\alpha\alpha_s^2) \sim 0.035 \text{ MeV}$$

$$\delta_5 : \mathcal{O}(\alpha_{bos}^2) \sim \mathcal{O}(\alpha_{bos})^2 \sim \mathbf{0.1} \text{ MeV}$$

2. Parametric prefactors

$$\delta_6 : \mathcal{O}(\alpha_{bos}^2) \sim \Gamma_Z\mathcal{O}(\alpha^2) \sim \mathbf{0.1\ MeV}$$

$$\delta_7 : \mathcal{O}(\alpha\alpha_s^2) - \mathcal{O}(\alpha_t\alpha_s^2) \sim \Gamma_Z\frac{\alpha n_q}{\pi}\alpha_s^2 \sim 0.29 \text{ MeV}$$

Total: $\delta\Gamma_Z = \sqrt{\sum_{i=1}^{5}\delta_i^2} \sim \mathbf{0.5}$ **MeV**

Future projections, Ayres: 1604.00406

	Measurement error			Intrinsic theory	
	ILC	CEPC	FCC-ee	Current	Future†
M_{W} [MeV]	3–4	3	1	4	1
Γ_{Z} [MeV]	0.8	0.5	**0.1**	0.5	**0.2**
R_b $[10^{-5}]$	14	17	6	15	7
$\sin^2\theta^{\ell}_{\mathrm{eff}}$	1	2.3	**0.6**	4.5	**1.5**

Table: Projected experimental and theoretical uncertainties for some electroweak precision pseudo-observables.

† Based on estimations for: $\mathcal{O}(\alpha^2_{bos})$,$\mathcal{O}(\alpha\alpha^2_s)$, $\mathcal{O}(N_f\alpha^2\alpha_s)$, $\mathcal{O}(N^2_f\alpha^3)$

New results for completing NNLO

Input parameters:

Parameter	Value	Parameter	Value
M_{Z}	91.1876 GeV	$m_{\mathrm{b}}^{\overline{\mathrm{MS}}}$	4.20 GeV
Γ_{Z}	2.4952 GeV	$m_{\mathrm{c}}^{\overline{\mathrm{MS}}}$	1.275 GeV
M_{W}	80.385 GeV	m_τ	1.777 GeV
Γ_{W}	2.085 GeV	$\Delta\alpha$	0.05900
M_{H}	125.1 GeV	$\alpha_{\mathrm{s}}(M_{\mathrm{Z}})$	0.1184
m_{t}	173.2 GeV	G_μ	1.16638×10^{-5} GeV^{-2}

The cherry on the 2-loops EWPOs cake: results for $\mathcal{O}(\alpha^2_{\rm bos})$ **[preliminary]**

	Γ_Z [MeV]	$\sigma^0_{\rm had}$ [pb]
$\mathcal{O}(\alpha)$	60.22	-48.86
$\mathcal{O}(\alpha\alpha_s)$	9.11	3.14
$\mathcal{O}(\alpha_t\alpha_s^2,\, \alpha_t\alpha_s^3,\, \alpha_t^2\alpha_s,\, \alpha_t^3)$	1.20	0.48
$\mathcal{O}(N_f^2\alpha^2)$	5.13	-1.03
$\mathcal{O}(N_f\alpha^2)$	3.04	9.09
$\mathcal{O}(\alpha^2_{\rm bos})$	**0.51**	**1.27**

Remark #1, so far:

Error estimation, bosonic NNLO ~ 0.1 MeV

Total error estimation for $\Gamma_Z \sim 0.5$ MeV

Remark #2:

FCC-ee$^{exper.\ error}(\Gamma_Z) \sim 0.1$ MeV

FCC-ee$^{theor.\ error}(\Gamma_Z) \sim 0.2$ MeV

Γ_i [MeV]	$\Gamma_e, \Gamma_\mu, \Gamma_\tau$	$\Gamma_{\nu_e}, \Gamma_{\nu_\mu}, \Gamma_{\nu_\tau}$	Γ_d, Γ_s	Γ_u, Γ_c	Γ_b	Γ_Z
$\mathcal{O}(\alpha)$	2.273	6.174	9.717	5.799	3.857	60.22
$\mathcal{O}(\alpha\alpha_s)$	0.288	0.458	1.276	1.156	2.006	9.11
$\mathcal{O}(\alpha_t\alpha_s^2,\, \alpha_t\alpha_s^3,\, \alpha_t^2\alpha_s,\, \alpha_t^3)$	0.038	0.059	0.191	0.170	**0.190**	1.20
$\mathcal{O}(N_f^2\alpha^2)$	0.244	0.416	0.698	0.528	0.694	5.13
$\mathcal{O}(N_f\alpha^2)$	0.120	0.185	0.493	0.494	0.144	3.04
$\mathcal{O}(\alpha^2_{\rm bos})$	**0.017**	**0.019**	**0.058**	**0.057**	**0.167**	**0.51**

Having this knowledge: we need **genuine** 3-loop vertex calculations!

1. Geometric series

$$\mathcal{O}(\alpha^3) - \mathcal{O}(\alpha_t^3) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)}\mathcal{O}(\alpha^2) \sim 0.26 \text{ MeV}$$

$$\mathcal{O}(\alpha^2\alpha_s) - \mathcal{O}(\alpha_t^2\alpha_s) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)}\mathcal{O}(\alpha\alpha_s) \sim 0.3 \text{ MeV}$$

$$\mathcal{O}(\alpha\alpha_s^2) - \mathcal{O}(\alpha_t\alpha_s^2) \sim \frac{\mathcal{O}(\alpha\alpha_s) - \mathcal{O}(\alpha_t\alpha_s)}{\mathcal{O}(\alpha)}\mathcal{O}(\alpha\alpha_s) \sim 0.23 \text{ MeV}$$

$$\mathcal{O}(\alpha\alpha_s^3) - \mathcal{O}(\alpha_t\alpha_s^3) \sim \frac{\mathcal{O}(\alpha\alpha_s) - \mathcal{O}(\alpha_t\alpha_s)}{\mathcal{O}(\alpha)}\mathcal{O}(\alpha\alpha_s^2) \sim 0.035 \text{ MeV}$$

$$\mathcal{O}(\alpha_{bos}^2) \sim \mathcal{O}(\alpha_{bos})^2 \sim \mathbf{0.1\ MeV} \longrightarrow [\mathbf{0.51}\text{ MeV}]$$

2. Parametric prefactors

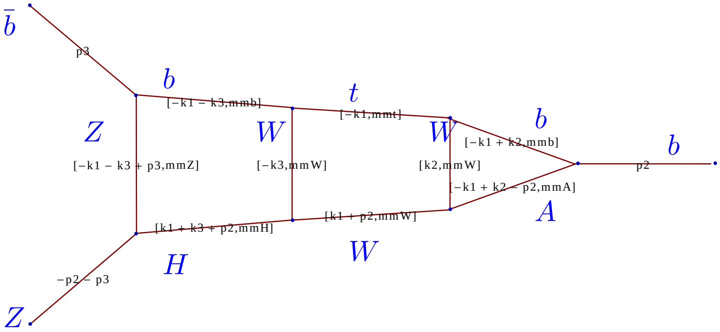
$$\mathcal{O}(\alpha_{bos}^2) \sim \Gamma_Z\mathcal{O}(\alpha^2) \sim \mathbf{0.1\ MeV}$$

$$\mathcal{O}(\alpha\alpha_s^2) - \mathcal{O}(\alpha_t\alpha_s^2) \sim \Gamma_Z\frac{\alpha n_q}{\pi}\alpha_s^2 \sim 0.29 \text{ MeV}$$

Total: $\delta\Gamma_Z \sim \mathbf{0.5}$ **MeV** $\longrightarrow$ **underestimated**

A complete zoo of heavy particles m_t, m_W, m_Z, m_H @NNNLO level

MB: ϵ^0**[8-dim]**, $1/\epsilon$**[7-dim]**; SD: ϵ^0**[8-dim]** , $1/\epsilon$**[7-dim]**;



At 2-loops up to three dimensionless parameters (all 4 at 3-loops):

$$\left\{ \frac{M_H^2}{M_Z^2}, \frac{M_W^2}{M_Z^2}, \frac{m_t^2}{M_Z^2}, \frac{(M_Z + i\varepsilon)^2}{M_Z^2} \right\}$$

Basic bookkeeping

$Z \to b\bar{b}$			
Number of topologies	1 loop	2 loops	3 loops
	1	14$\rightarrow^{(\mathbf{A})}$ 7 $\rightarrow^{(\mathbf{B})}$ **5**	211 $\rightarrow^{(\mathbf{A})}$ 84 $\rightarrow^{(\mathbf{B})}$ **50**
Number of diagrams	15	2383$\rightarrow^{(\mathbf{A},\mathbf{B})}$ **1114**	490387 $\rightarrow^{(\mathbf{A},\mathbf{B})}$**120187**
Fermionic loops	0	150	17580
Bosonic loops	15	964	102607
Planar diagrams	1T/15D	4T/981D	35T/84059D
Non-planar diagrams	0	1T/133D	15T/36128D

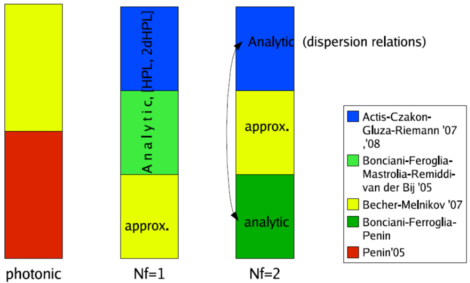
Table: Some statistical overview for $Z \to b\bar{b}$ multiloop studies. At 3 loops there are in total almost half a million of diagrams present. After basic refinements (A) and (B) about 10^5 genuine 3-loop vertex diagrams remain. In (A) tadpoles and products of lower loops are excluded, in (B) symmetries of topologies are taken into account.

Several reasons to stay optimistic in "microscoping" higher order calculations

1. **Steady** progress in numerical calculations, methods and tools;
2. Lessons from the past (LEP, LHC,...) - anticipated SM predictions improved considerably - sometimes even several times after experiments took off;
3. Often problems can be attacked from different perspectives (it is needed for independent confirmations);

Present situation, virtual NNLO QED

Bhabha scattering, 10 years ago



To substantiate my claim even more: An example of effective weak mixing angle $\sin^2\theta_{\rm eff}^{\rm b}$

- The standard model prediction for the effective weak mixing angle can be written as

$$\sin^2\theta_{\rm eff}^{\rm b} = \left(1 - \frac{M_W^2}{M_Z^2}\right)(1 + \Delta\kappa_{\rm b})$$

- The bosonic electroweak two-loop corrections amount to

$$\boxed{\Delta\kappa_{\rm b}^{(\alpha^2,{\rm b}os)} = -0.9855 \times 10^{-4}}$$

DFGRU, Phys.Lett. B762 (2016) 184

Collection of radiative corrections: Full stabilization at 10^{-4}!

Order	Value $[10^{-4}]$	Order	Value $[10^{-4}]$
α	468.945	$\alpha_t^2\alpha_s$	1.362
$\alpha\alpha_s$	-42.655	α_t^3	0.123
α^2_{ferm}	3.866	$\alpha_t\alpha_s^2$	-7.074
α^2_{bos}	**-0.986**	$\alpha_t\alpha_s^3$	-1.196

$\pm 0.001 \xrightarrow{!}$

Table: *Comparison of different orders of radiative corrections to* $\Delta\kappa_{\mathrm{b}}$.

Input Parameters: M_Z, Γ_Z, M_W, Γ_W, M_H, m_t, α_s *and* $\Delta\alpha$

- one-loop contributions [Akhundov, Bardin, Riemann, 1986] [Beenakker, Hollik, 1988]
- two-loop fermionic contributions [Awramik, Czakon, Freitas, Kniehl, 2009]
- two-loop bosonic contributions [Dubovyk, Freitas, JG, Riemann, Usovitsch, 2016]

Partial higher-order corrections

$\mathcal{O}(\alpha_{\mathrm{t}}\alpha_{\mathrm{s}}^2)$	Avdeev: 1994, Chetyrkin: 1995
$\mathcal{O}(\alpha_{\mathrm{t}}\alpha_{\mathrm{s}}^3)$	Schroder: 2005, Chetyrkin: 2006, Boughezal: 2006
$\mathcal{O}(\alpha^2\alpha_{\mathrm{t}})$ and $\mathcal{O}(\alpha_{\mathrm{t}}^3)$	vanderBij: 2000, Faisst: 2003

MB and SD methods are very much complementary!

- MB works well for hard threshold, on-shell cases, not many internal masses (more IR); SD more useful for integrals with many internal masses
  $\longrightarrow$ talks by Evgen, Johann and Sophia;
  $\longrightarrow$ JG, Kajda, Riemann, Yundin, EPJC'11; JG in PoS-LL2016 & DFGRU in PLB'16.

Now realized in real application for EWPOs:

> 10^{-8} accuracy achieved for **any** self-energy and vertex Feynman integral with one of the methods.

Cuhre package [Thomas Hahn] - not a Monte Carlo method, crucial for MB with dim$\leq$5.

What is not covered in this workshop

plenty of things..., just one example

- R. N. Lee and K. T. Mingulov,
  "**DREAM**, a program for arbitrary-precision computation of dimensional recurrence relations solutions, and its applications", arXiv:1712.05173 [hep-ph];
- ...

Collective effort needed!

Conclusions for next years

- Strong demand from FCC-ee to the theory on precision;
- We need to go beyond NNLO: $\mathcal{O}(\alpha\alpha_s^2), \mathcal{O}(N_f\alpha^2\alpha_s), \mathcal{O}(N_f^2\alpha^3)$;
- 1. **We know how** to do it;
  2. and **we have appropriate tools;**
- To be on the safe side, we would like to have **at least 2 independent calculations**;
- Still, a lot work ahead, for success and efficiency, **we need also the steady progress in (semi)analytical approaches** in multiloop calculations.

Let us hope that this mini-workshop will be right way in proper direction!

Freeman Dyson:

"New directions in science are launched by new tools much more often than by new concepts.

The effect of a concept-driven revolution is to explain old things in new ways.

The effect of a **tool-driven revolution** *is to discover new things that have to be explained"*

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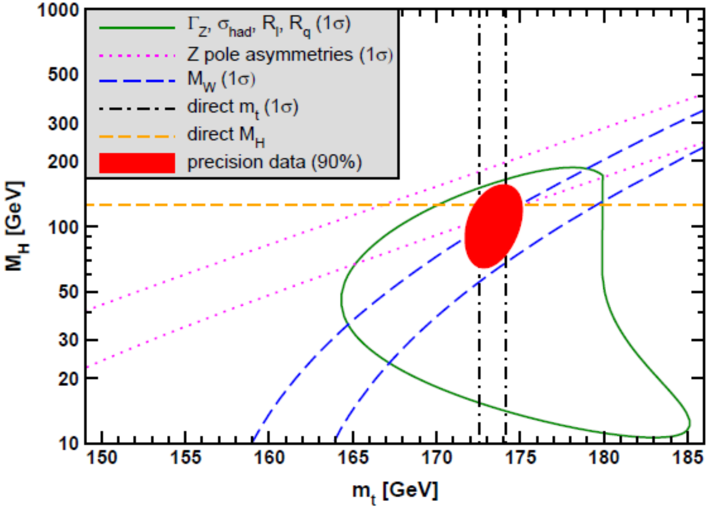

Direct measurements:

$M_H = 125.09 \pm 0.24$ GeV

$m_t = 173.34 \pm 0.81$ GeV

Indirect prediction:

$M_H = 126.1 \pm 1.9$ GeV

(with LHC BRs)

$M_H = 96^{+22}_{-19}$ GeV

(w/o LHC data)

$m_t = 176.7 \pm 2.1$ GeV