The Bhabha Project

Janusz Gluza
Katowice, Poland

in honour of the 60th birthday of Tord Riemann

from the beginning Tord tries to understand the world

Tord-3yearsold-damaged-equipment.jpg (photos thanks to Sabine R.)

Tord is 60 years young,

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- SPIRES: 218 papers by Tord Riemann

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- promoted 2 MSc students, 4 PhD student (the next one is waiting in a queue)

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- SPIRES: 218 papers by Tord Riemann
- promoted 2 MSc students, 4 PhD student (the next one is waiting in a queue)
- over 5000 citations, many (tens) of collaborators from many countries, working in many countries (Dubna, Geneva, Münich)

working in Geveva, 1992

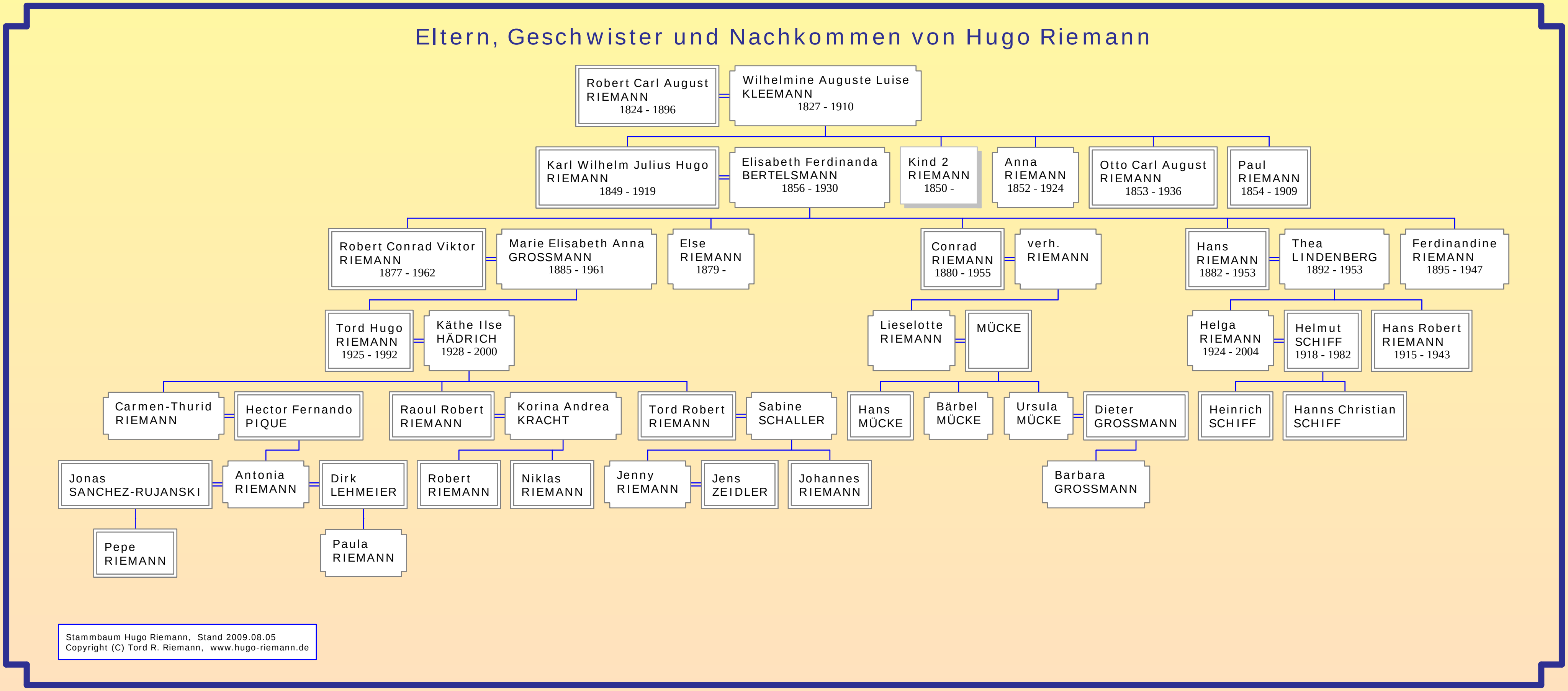
India, three Amigos

China and Tord

(scientific) fields of interest

- LHC physics
- Loop calculations
- NNLO Bhabha scattering in QED
- Higher order corrections in the electroweak Standard Model (and some extensions)
- Phenomenology of elementary particles; precision tests of fundamental interactions
- Linear Collider physics; Higgs, WW, ZZ, 2f and 4f production in e+e- scattering
- HERA physics; QED and electroweak corrections for deep inelastic scattering at HERA
- See also the homepages on Ambre, Bhabha Scattering, DIANA, aITALC, ZFITTER, HECTOR

Family tree - another research



- Awarded January 19, 2001 by the Scientific Council of the JINR Dubna for the project "Theoretical support of experiments at the Z resonance on precision tests of the standard model (Project ZFITTER)"

Рецензия

на цикл работ "Теоретическая поддержка экспериментов на Z резонансе по прецизионной проверке Стандартной модели, проект ZEITTER", авторы: Ю.Д. Бардин, М.С. Биленький, М.Джак, Л.В. Калиновская, А.Г. Ольшевский, С. Риманн, Т. Риманн, П.Х. Христова, выдвинутых на премию ОИЯИ'2000 по категории научно-исследовательских теоретических работ.

ON JANUARY 19, 2001, THE SCIENTIFIC COUNCIL
OF THE JOINT INSTITUTE FOR NUCLEAR RESEARCH
AWARDED

THE FIRST PRIZE
OF THE JOINT INSTITUTE
FOR NUCLEAR RESEARCH

to

Tord RIEMANN

FOR THE WORK «THEORETICAL SUPPORT
OF EXPERIMENTS AT THE *Z* RESONANCE ON PRECISION TESTS
OF THE STANDARD MODEL (PROJECT **ZFITTER**)»

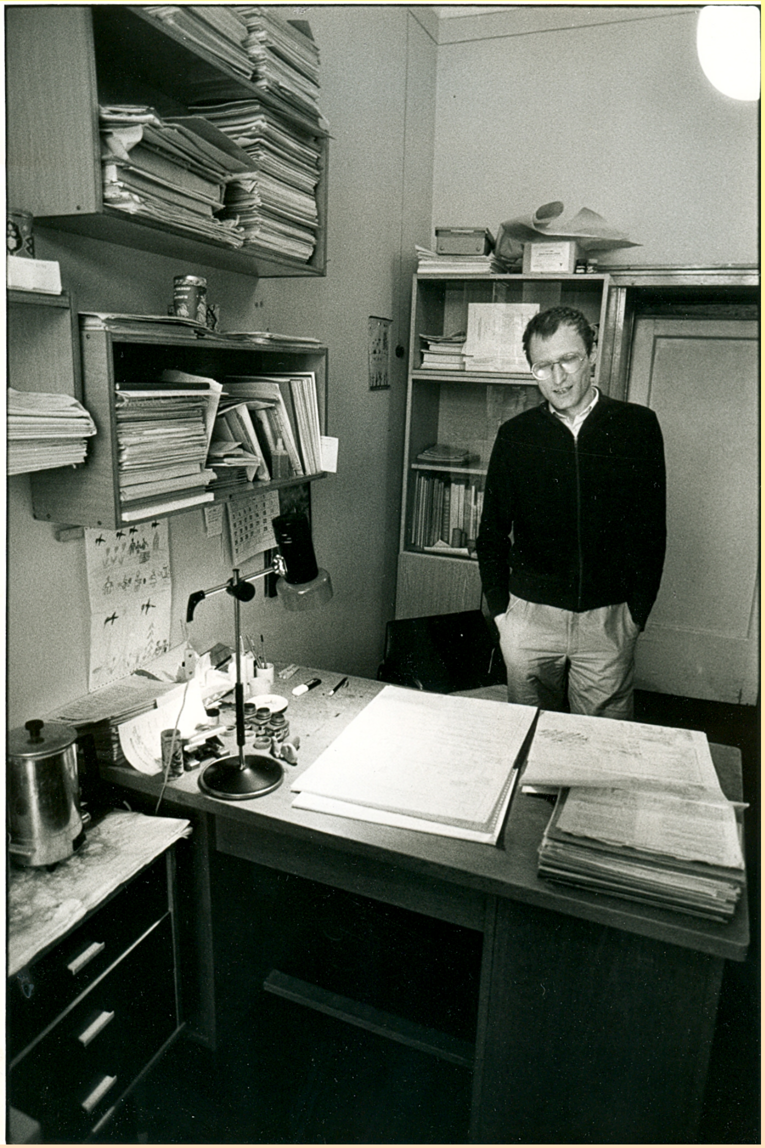
Chairman of the Scientific Council
of the Joint Institute
for Nuclear Research,
Director — V.G.Kadyshevsky

Vice-Director — A.N.Sissakian

Vice-Director — Ts.Vylov

Dubna
January 19, 2001
№ 2632

in Dubna



tough winters deep in Russia

numerical and symbolic packages

numerical and symbolic packages

1. ZFITTER - A Program for the Semi-Analytic Calculation of Predictions for the Process $e^+e^- \to 2f$
2. topfit - A Program for the Semi-Analytic Calculation of Predictions for the Process $e^+e^- \to t\bar{t}\ (+photon)$
3. GENTLE/4fan - A Program for the Semi-Analytic Calculation of Predictions for the Process $e^+e^- \to 4f$
4. Predictions for Anomalous tau+ tau- gamma Production at LEP 1
5. HECTOR 1.00 - A program for the calculation of QED, QCD and electroweak corrections to ep and lN deep inelastic neutral and charged current scattering
6. SMATASY: a program for the model independent description of the Z resonance
7. AMBRE - Automatic Mellin-Barnes REpresentation, in Mathematica
8. CSectors - numerical calculation of multiloop tensor integrals in Euclidean region by sector decomposition , in Mathematica.

main collaborators

- Dima Bardin (66 papers)

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- Jochem Fleischer (24 papers)

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- J.G. (22 papers)
- Wolfgang Hollik (15 papers)
- Sabine Riemann (5 papers)


Indian Heroes
..iloveindia

- Homi Jehangir Bhabha, 1909-1966

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- father of the Indian atomic energy program

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- father of the Indian atomic energy program
- "The scattering of positrons by electrons with exchange on Dirac's theory of the positron", Proc.Roy.Soc.Lond.A154:195-206,1936.

The Scattering of Positrons by Electrons with Exchange on Dirac's Theory of the Positron

By H. J. BHABHA, Ph.D., Gonville and Caius College

(*Communicated by R. H. Fowler, F.R.S.—Received October* 20, 1935)

It has been shown by Mott† that exchange effects play a considerable part in the collision and consequent scattering of one electron by another. Mott's original calculation was non-relativistic, and there the exchange effect vanishes when the two electrons have their spins pointing in opposite directions. Møller‡ later developed relativistically invariant expressions for the collision of two charged particles with spin, and it may be seen directly from Møller's general formula for the collision cross-section that, in the collision of two identical particles, the effect of exchange does not in general vanish even when the two colliding particles initially have their spins pointing in opposite directions. It tends however to zero in this case as the relative velocity of the particles becomes small compared to *c*, the velocity of light, in agreement with the calculation of Mott.

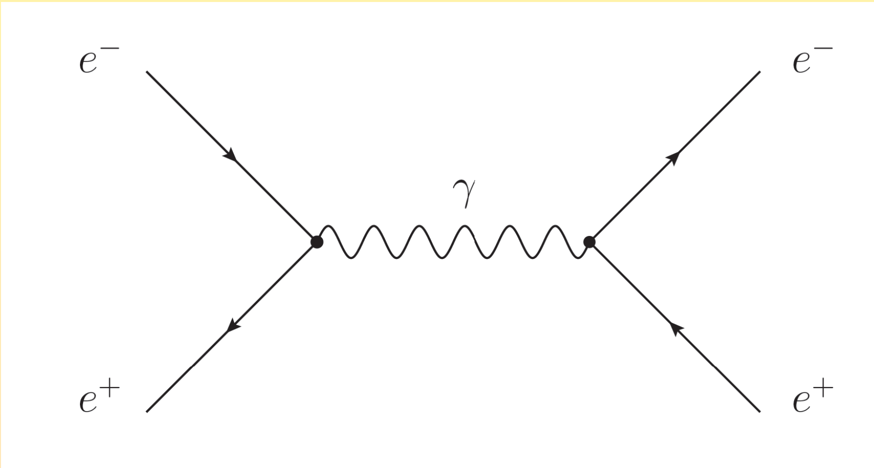
The effect of exchange in the general relativistic case will still be considerable if one of the two electrons be initially (and therefore finally) in a state of negative energy. (If one of the electrons be initially in a negative energy state, then it follows from the conservation of energy

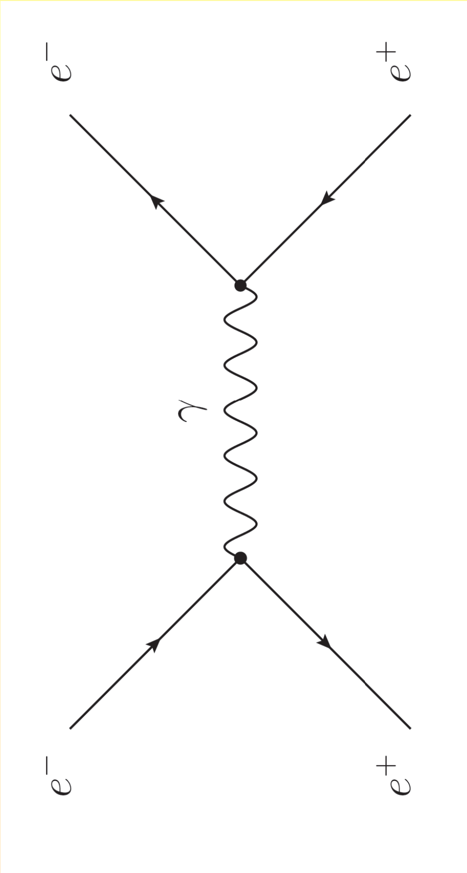
† 'Proc. Roy. Soc.,' A, vol. 126, p. 259 (1930).
‡ 'Ann. Physik,' vol. 14, p. 531 (1932).

where we have inserted in expressions like (11) the values of $E_1'^*$, $\mathbf{p}_1'^*$, etc., in terms of E^*, p^*, $\mathbf{p}'^*$. The spurs in (14) are easily evaluated if we remember that the spurs of all the Dirac matrices and their products are zero, excepting that of the unit matrix.

We get finally for the differential effective cross-section dQ^* for the scattering of the electron through an angle between θ^* and $\theta^* + d\theta^*$ in the system L^* the expression

$$dQ^* = \frac{\pi}{8}\frac{e^4}{m^2c^4\gamma^{*2}}\Big[\frac{1}{(\gamma^{*2}-1)^2\sin^4\frac{1}{2}\theta^*}\{1+4(\gamma^{*2}-1)\cos^2\tfrac{1}{2}\theta^* + 2(\gamma^{*2}-1)^2(1+\cos^4\tfrac{1}{2}\theta^*)\} + \frac{1}{\gamma^{*4}}\{3+4(\gamma^{*2}-1)+(\gamma^{*2}-1)^2(1+\cos^2\theta^*)\} - \frac{1}{\gamma^{*2}(\gamma^{*2}-1)\sin^2\frac{1}{2}\theta^*}\{3+4(\gamma^{*2}-1)(1+\cos\theta^*) + (\gamma^{*2}-1)^2(1+\cos\theta^*)^2\}\Big].\sin\theta^*\,d\theta^*. \quad (15)$$


e^-
e^-
γ
e^+
e^+


e^-
e^+
γ
e^-
e^+

Luminosity of a collider depends on the machine (ϵ) and the beam (L): all is complicated (not mentioning errors' estimation)

$$\frac{dN}{dt} = \epsilon \cdot \mathcal{L}(t), \quad N = \epsilon \cdot \sigma \int dt \mathcal{L}(t) = \epsilon \cdot \sigma \cdot \mathcal{L} \tag{1}$$

better is to choose a well known process:

$$\mathcal{L} = \frac{1}{\epsilon \cdot \sigma_{Bhabha}} N_{Bhabha}$$

then we may determine any cross section in (1)

$$\frac{\mathsf{d}\sigma_0^{\mathrm{Bhabha}}}{\mathsf{d}\Omega_-} = \frac{\alpha^2}{4s} \left(\frac{3+c^2}{1-c}\right)^2 + O\left(\frac{m_e^2}{s}\right),$$

where

$$s = (p_- + p_+)^2, \qquad c = \cos\theta_-.$$

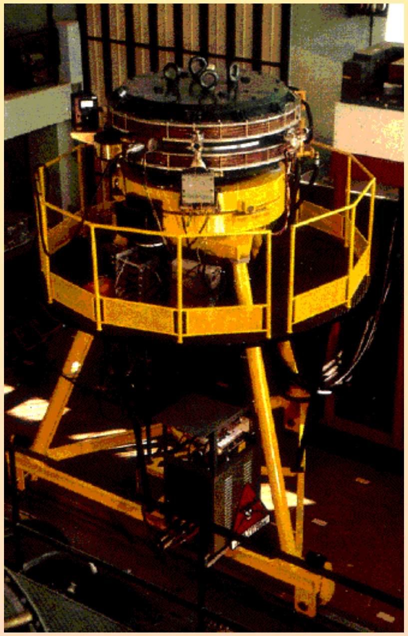
$$\sigma \sim \frac{1}{\theta_-^3}(!)$$

measurements at miliradians (1-3 degrees)

first colliders

- 1960's: Adone in Frascati, ACO at Orsay, VEPP-II at Novosibirsk, $1 \leq \sqrt{s} \leq 3\, GeV$

  Ada, Bruno Touschek

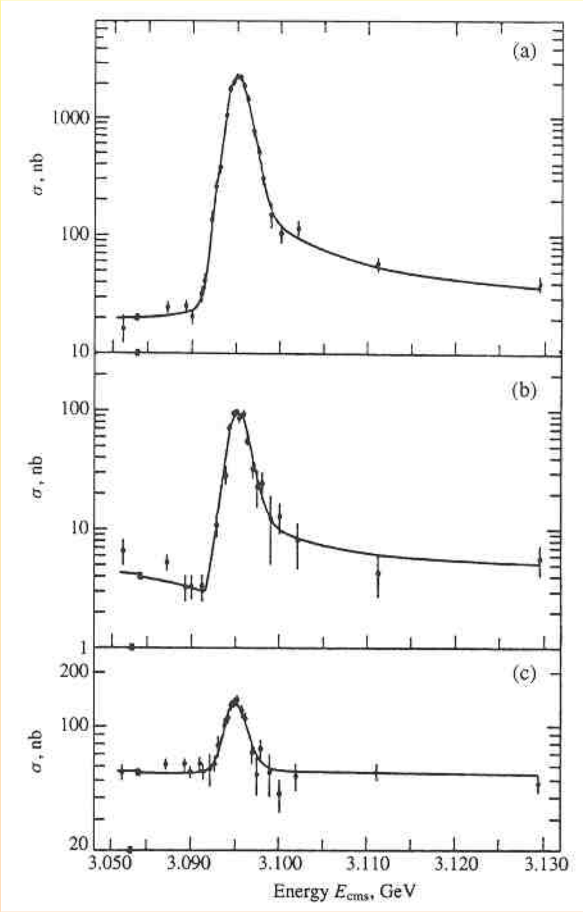


sometimes we should pray, maybe

I-guess-end-of-year-party-1976.jpg

SPEAR, SLAC, charm discovery

Gerson Goldhaber, Martin Perl, and Burton Richter, Nobel Prize 1976



- next step, LEP and SLC

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- now $\left[ILC \right]$ and meson factories

- 23.06.2011: 295 papers in SPIRES with the title "Bhabha"

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- 22 of them by Tord (of 218 papers in total)

Z. Phys. C – Particles and Fields 49, 485–490 (1991)

Zeitschrift für Physik C Particles and Fields

Bhabha scattering with higher order weak loop corrections

D. Bardin[1], W. Hollik[2], T. Riemann[2,]*

[1] Joint Institute for Nuclear Research, Dubna, Head Post Office, P.O. Box 79, SU-101000 Moscow, USSR
[2] Max-Planck-Institut für Physik and Astrophysik, Werner-Heisenberg-Institut für Physik, P.O. Box 40 12 12, W-8000 München, Federal Republic of Germany

Received 10 August 1990

Abstract. We present a compact incorporation of the weak loop effects in the Bhabha cross section with the accuracy required by the present precision experiments. Special care is taken of the leading 2-loop effects from a heavy top quark which are of the type $\alpha^2(m_t/M_Z)^4$. We find very satisfactory agreement between two independent calculations which are based on different gauges and different renormalizations. The compact formulation in terms of form factors allows a straight-forward implementation into analytic or Monte Carlo QED calculations.

and of fermion pair production [12, 13], the electroweak predictions for Bhabha scattering deserve definite improvements in order to reach uncertainties for the angular distribution below 1%. In view of some recent phenomenological studies (e.g. [7, 14, 15]) which propose to use wide angle Bhabha scattering for electroweak high precision measurements of e.g. Γ_Z or $\sin^2\theta_W$, there could be even need of an accuracy of about 0.3% for cross section predictions.

The aim of the present article is to obtain the electroweak (non-QED) radiative corrections to Bhabha-

Table 3. The integrated Bhabha cross section σ_T in nbarn for two angular intervals $|\cos\theta| < x_m$, $x_m = 0.9659, 0.7071$, as function of $\sqrt{s}$, m_t, M_H for $M_Z = 91.16$ GeV. Upper rows: DZ, lower rows: H (all massess in GeV)

m_t	$\sqrt{s}$ / M_H	89 / 0.9659	0.7071	91.16 / 0.9659	0.7071	93 / 0.9659	0.7071
100	100	2.853	0.5839	3.651	1.352	1.731	0.3459
		2.853	0.5839	3.651	1.352	1.731	0.3459
150	25	2.858	0.5864	3.652	1.352	1.734	0.3481
		2.858	0.5865	3.649	1.352	1.734	0.3480
150	100	2.858	0.5865	3.653	1.353	1.734	0.3482
		2.858	0.5866	3.652	1.353	1.734	0.3482
150	1000	2.856	0.5853	3.655	1.355	1.732	0.3471
		2.856	0.5853	3.652	1.355	1.733	0.3470
200	100	2.865	0.5902	3.657	1.355	1.738	0.3515
		2.865	0.5904	3.654	1.355	1.738	0.3514
250	100	8.874	0.5949	3.662	1.357	1.744	0.3560
		2.874	0.5954	3.658	1.358	1.744	0.3556

Fleischer, Lorca, TR, 2004

$e^+e^- \to e^+e^- \qquad \sqrt{s} = 500$ GeV			
$\cos\theta$	$\left[\frac{d\sigma}{d\cos\theta}\right]$ Born /pb	$\left[\frac{d\sigma}{d\cos\theta}\right]$ B+1-loop /pb	Model
$-.9000$	0.216 998	0.144 359	EWSM
$-.9000$	0.523 873	0.387 798	QED
$-.5000$	0.261 360	0.181 086	EWSM
$-.5000$	0.611 600	0.471 451	QED
0.0000	0.598 142	0.431 573	EWSM
0.0000	$0.117\ 253 \cdot 10^1$	0.916 946	QED
0.5000	$0.421\ 272 \cdot 10^1$	$0.320\ 045 \cdot 10^1$	EWSM
0.5000	$0.550\ 440 \cdot 10^1$	$0.435\ 535 \cdot 10^1$	QED
0.9000	$0.189\ 160 \cdot 10^3$	$0.150\ 885 \cdot 10^3$	EWSM
0.9000	$0.189\ 118 \cdot 10^3$	$0.152\ 861 \cdot 10^3$	QED
0.9900	$0.206\ 555 \cdot 10^5$	$0.170\ 576 \cdot 10^5$	EWSM
0.9900	$0.206\ 381 \cdot 10^5$	$0.170\ 818 \cdot 10^5$	QED
0.9990	$0.208\ 236 \cdot 10^7$	$0.176\ 139 \cdot 10^7$	EWSM
0.9990	$0.208\ 242 \cdot 10^7$	$0.176\ 190 \cdot 10^7$	QED
0.9999	$0.208\ 429 \cdot 10^9$	$0.180\ 172 \cdot 10^9$	EWSM
0.9999	$0.208\ 430 \cdot 10^9$	$0.180\ 178 \cdot 10^9$	QED

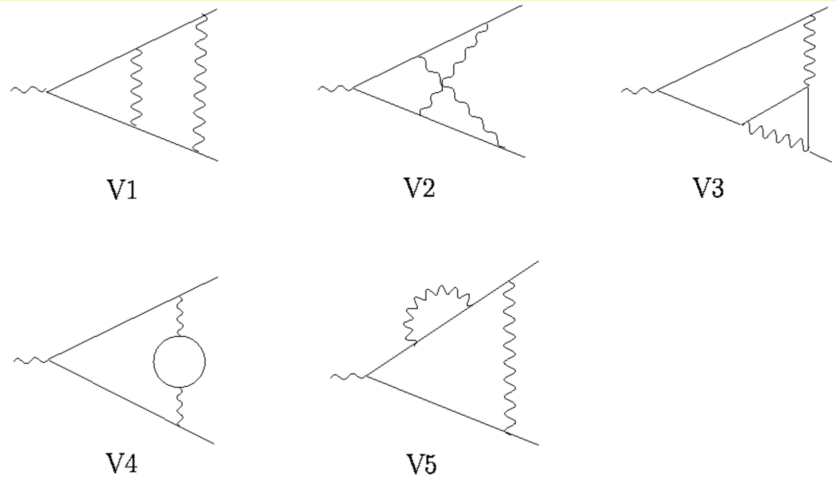
massive QED NNLO corrections

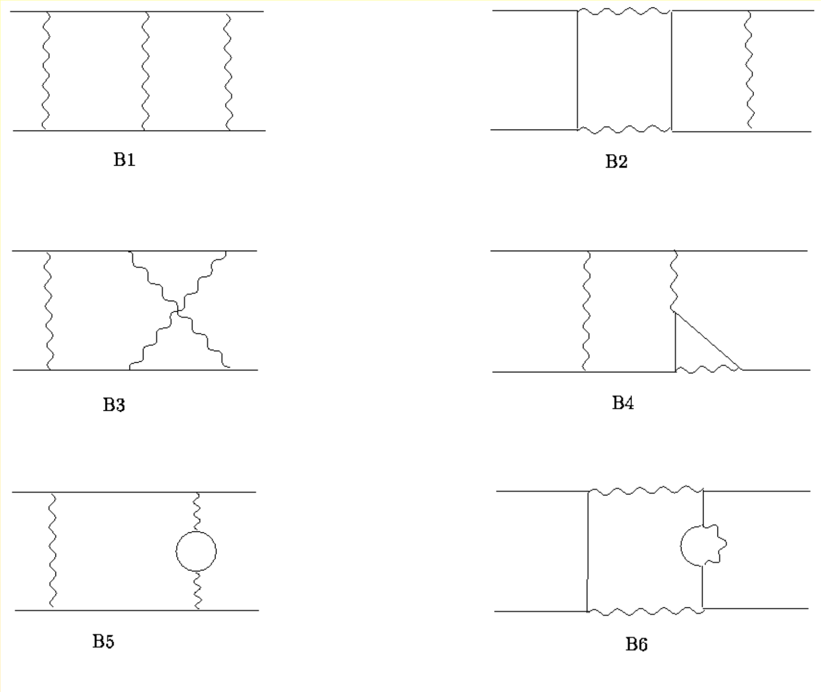
- NNLO leptonic and hadronic corrections to Bhabha scattering and luminosity monitoring at meson factories.

  C.Carloni Calame, H. Czyz, J. Gluza, M. Gunia, G. Montagna, O. Nicrosini, F. Piccinini, T. Riemann, M. Worek,

  June 2011. 38pp.

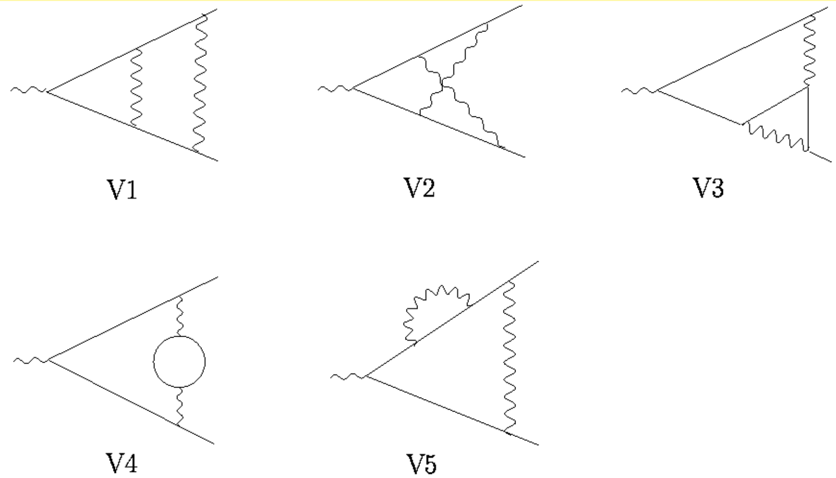
  e-Print: arXiv:1106.3178 [hep-ph]

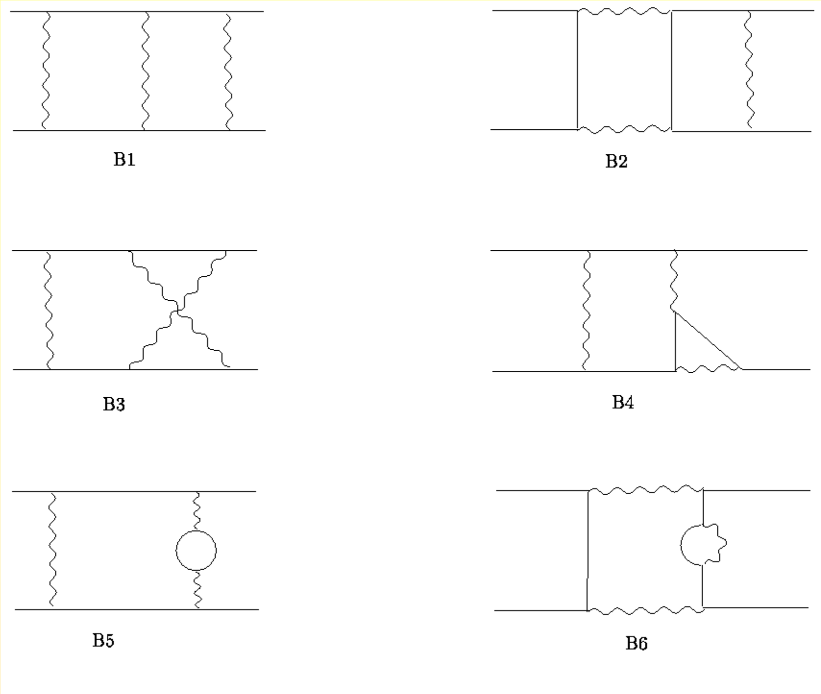




2-loop boxes for Bhabha process

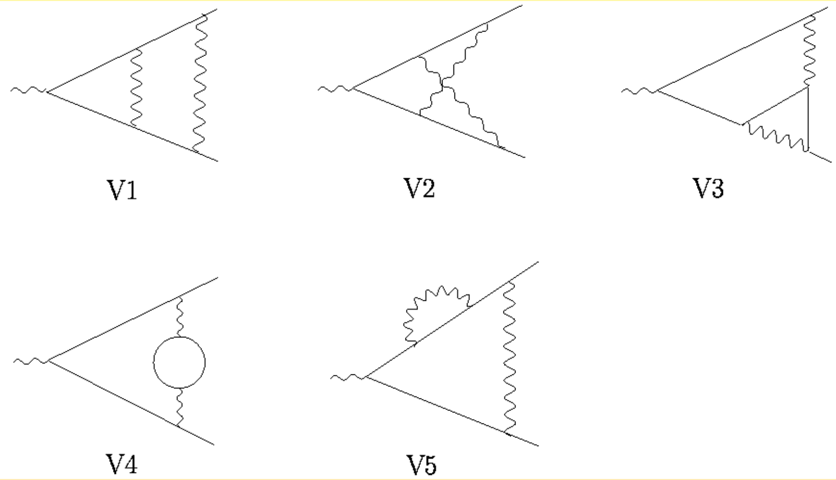
- SE loop insertions (without photonic line) are so called fermionic diagrams, rest represents photonic.

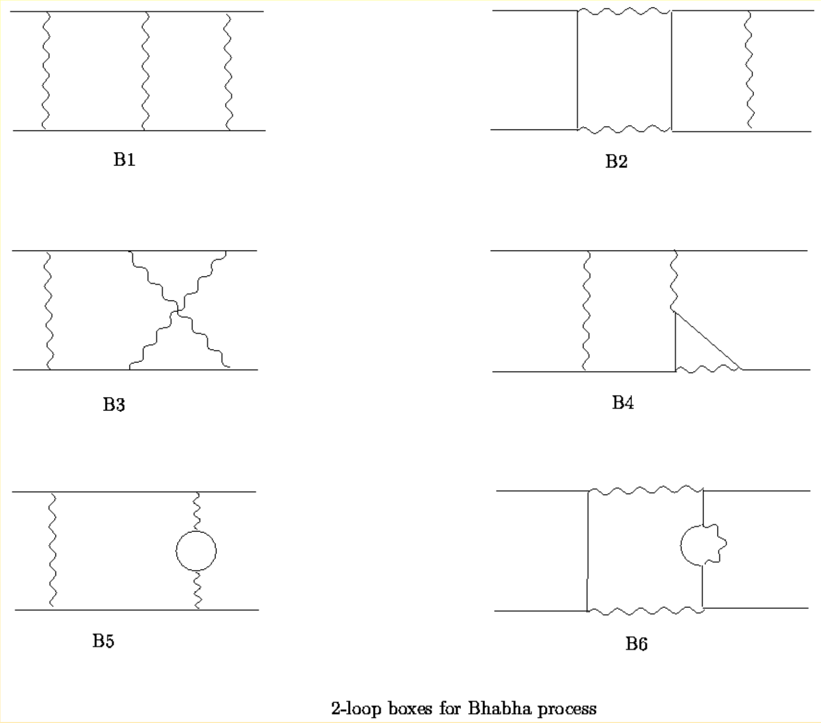




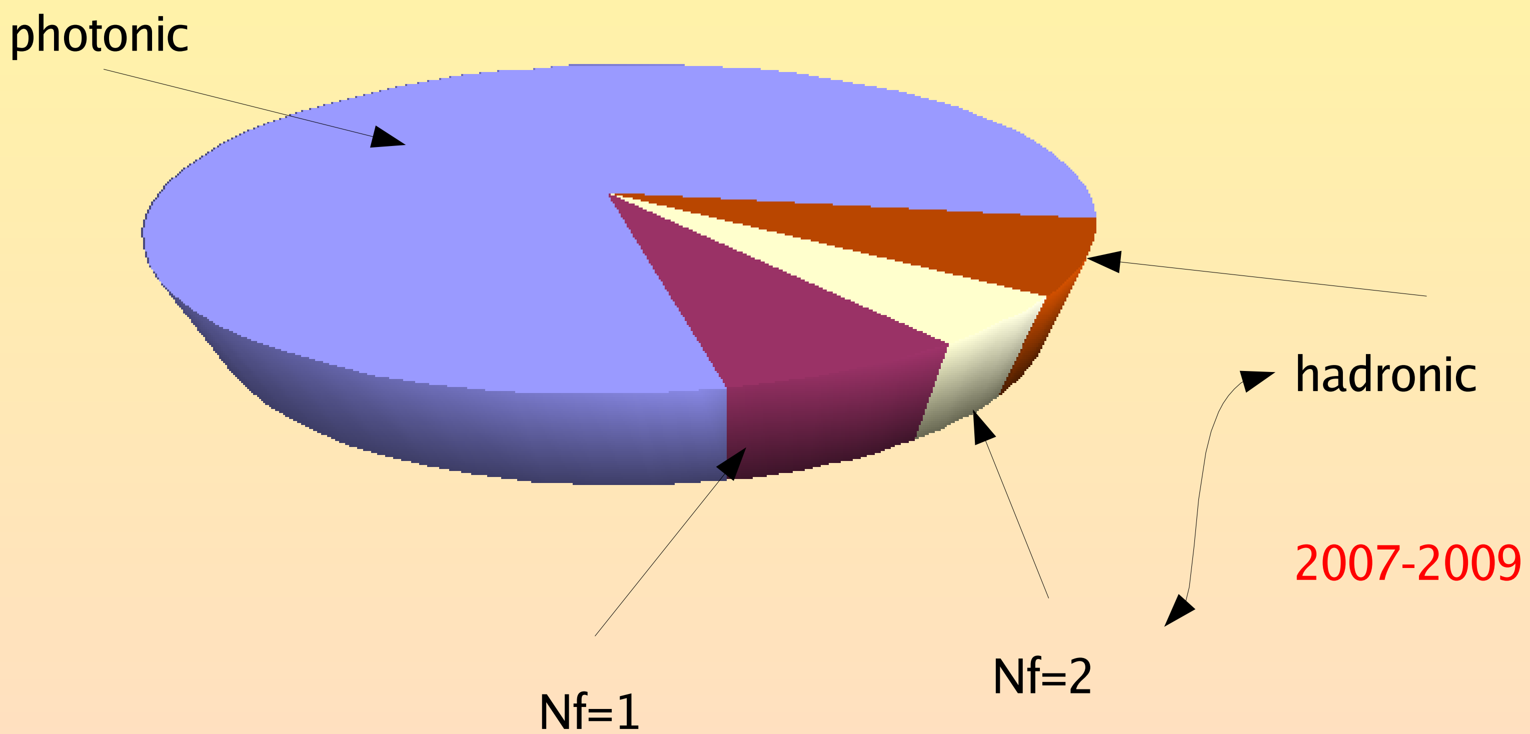
2-loop boxes for Bhabha process

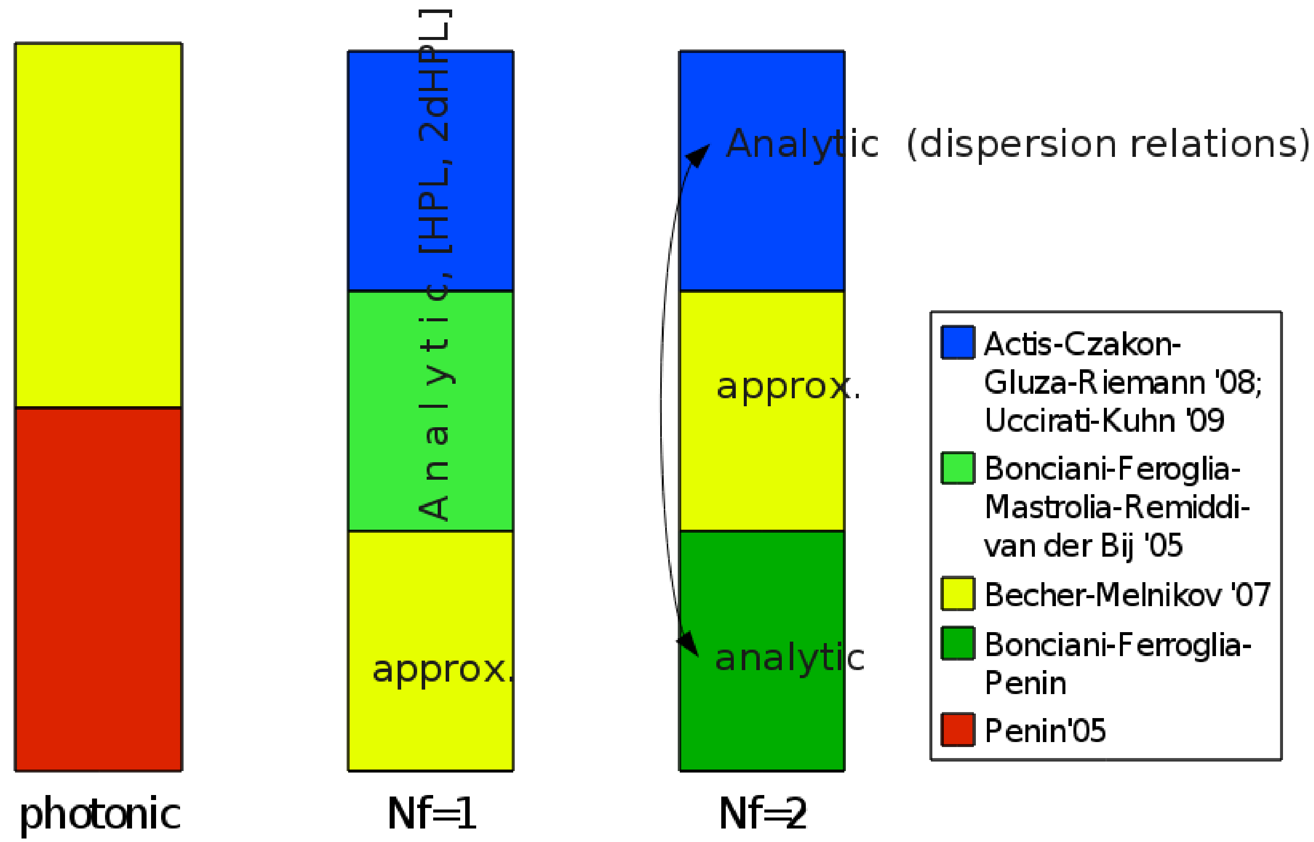
- SE loop insertions (without photonic line) are so called fermionic diagrams, rest represents photonic.
- Closed fermionic loop can be muon, tau, top or light quarks.





- SE loop insertions (without photonic line) are so called fermionic diagrams, rest represents photonic.
- Closed fermionic loop can be muon, tau, top or light quarks.
- In general, box B5 is a 4-scale problem: $m_e, m_f, s, t(u)$.


photonic
hadronic
2007-2009
Nf=2
Nf=1


Analytic, [HPL, 2dHPL]
Analytic (dispersion relations)
approx.
approx.
analytic
photonic
Nf=1
Nf=2
Actis-Czakon-Gluza-Riemann '08; Uccirati-Kuhn '09
Bonciani-Feroglia-Mastrolia-Remiddi-van der Bij '05
Becher-Melnikov '07
Bonciani-Ferroglia-Penin
Penin'05

Photonic corrections (no fermion loop)

- $$\frac{d\sigma_2}{d\sigma_0} = \underbrace{\mathsf{A}\ \ln^2\left(\frac{s}{m_e^2}\right) + \mathsf{B}\ \ln\left(\frac{s}{m_e^2}\right)}_{\text{collinear}} + \underbrace{\mathsf{C}}_{\text{non collinear}}$$

- [Arbuzov,Kuraev,Shaikhatdenov '98] [Glover,Tausk,van der Bij '01] $\Rightarrow$ A, B

- [Penin '05] $\Rightarrow$ C=C$_0$ + $\mathcal{O}(m_e^2/s)$ from massless result

- Result of A. Penin confirmed by independent calculation [Becher-Melnikov '07]

Heavy Fermions with IR Matching excluding $\mathcal{O}(m_e^2/s)$

[Penin '05]: photonic corrections follow from a change in the regularization scheme

$$\mathcal{M}(m=0, \underbrace{1/\epsilon}_{\text{IR and collinear}}) \Rightarrow \mathcal{M}(\underbrace{\ln(s/m_e^2)}_{\text{collinear}}, \underbrace{\ln(\lambda^2/m_e^2)}_{\text{IR}}) + \delta\mathcal{M}$$

[Mitov-Moch '06]: for $2 \to n$ QED/QCD scattering process

$$\mathcal{M}(m \neq 0) = \prod_j \mathcal{Z}_j^{1/2} \mathcal{M}(m=0) \quad \mathcal{Z}_j = \frac{F_D(Q^2, m \neq 0)}{F_D(Q^2, m = 0)}$$

[Becher-Melnikov '07]: in QED assuming $m_e^2 << m_f^2 << s, t, u$ heavy fermions can be included

$$\mathcal{M}(m \neq 0) = \prod_j \mathcal{Z}_j^{1/2} \underbrace{\mathcal{S}(m_f)}_{\text{process dependent}} \mathcal{M}(m=0)$$

$N_f = 1$ NNLO massive QED Corrections

Fermionic corrections: electrons in SE loops

- [Bonciani,Ferroglia,Mastrolia,Remiddi,van der Bij '05]

  $\Rightarrow$ electron loops with full m_e dependence (original)

- [Actis, JG, Czakon, Riemann '07]

  $\Rightarrow$ electron loops with full m_e dependence (recalculated)

- [Becher-Melnikov '07]

  $\Rightarrow$ electron loops $m_e^2 << s, t$ (and in small electron limit)

Evaluation of the MIs Electron-Loop Case

MIs evaluated using the method of differential equations

[Kotikov '91] [Remiddi '97] [Caffo,Czyz,Laporta,Remiddi '98]

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Three-scale problem: s, t, m_e

$$x = \frac{\sqrt{s} - \sqrt{s - 4m_e^2}}{\sqrt{s} + \sqrt{s - 4m_e^2}} \qquad y = \frac{\sqrt{4m_e^2 - t} - \sqrt{-t}}{\sqrt{4m_e^2 - t} + \sqrt{-t}}$$

MIs: HPLs [Remiddi,Vermaseren '99] and 2dHPLs [Gehrmann,Remiddi '00]

$N_f = 2$ NNLO leptonic massive QED Corrections

First results:

- [Becher-Melnikov '07]

  $\Rightarrow m_e^2 << m_f^2 << s, t, u$

- [Actis, JG, Czakon, Riemann '07]

  $\Rightarrow m_e^2 << m_f^2 << s, t, u$

- [Bonciani, Ferroglia, Penin '07]

  $\Rightarrow$ here: $m_e^2 << m_f^2, s, t, u$

Finally, yet another approach (dispersion relations, hadrons can be treated in a consistent way!)

- [Actis, JG, Czakon, Riemann '08]

  $\Rightarrow m_e^2 << m_f^2, s, t, u$

I: Evaluation of the MIs Heavy Fermions

Four-scale problem: s, t, m_e, m_f $\rightarrow$ new heavy-fermion scale

- Exploit the hierarchy of scales $m_e^2 << m_f^2 << s, t, u$
- Evaluate the MIs neglecting $\mathcal{O}(m_e^2/m_f^2)$, $\mathcal{O}(m_e^2/s)$, $\mathcal{O}(m_f^2/s)$

Mellin-Barnes method [Smirnov '99] [Tausk '99] efficient

- $\mathcal{I}^{2L} \sim \underbrace{\int dz \left(\frac{m_e^2}{m_f^2}\right)^{z+\epsilon} \frac{\prod \Gamma_i(z,\epsilon)}{\prod \Gamma_j(z,\epsilon)}}_{\text{integral}} \Rightarrow \underbrace{\sum_k c_k \left(\frac{m_e^2}{m_f^2}\right)^k}_{\text{sum}}$

- MB representations by AMBRE [Kajda, JG, Riemann '07]

- analytical continuation in ϵ with `MB` [Czakon '05]

- sums with `XSummer` [Moch,Uwer '05]

Example: box MIs, $L_m(x) = \ln(-m^2/x)$ and $R = m^2/M^2$

$$
\begin{aligned}
\texttt{B[x,y]} &= \frac{m^{-4\epsilon}}{x}\Big\{\frac{1}{\epsilon^2}L_m(x) + \frac{1}{\epsilon}\Big(-\zeta_2 + 2L_m(x) + \frac{1}{2}L_m^2(x) + L_m(x)L_m(y)\Big) \\
&- 2\zeta_2 - 2\zeta_3 + 4L_m(x) + L_m^2(x) + \frac{1}{3}L_m^3(x) - 4\zeta_2 L_m(y) \ldots
\end{aligned}
$$

Zeta[2] = 1.6449340668482264364724151666 5

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&- \quad 2\zeta_2 - 2\zeta_3 + 4L_m(x) + L_m^2(x) + \frac{1}{3}L_m^3(x) - 4\zeta_2 L_m(y) \ldots
\end{aligned}
$$

Zeta[2] = 1.64493406684822643647241516665

e-Print: arXiv:1106.3178 [hep-ph]

NNLO leptonic and hadronic corrections to Bhabha scattering
and luminosity monitoring at meson factories.

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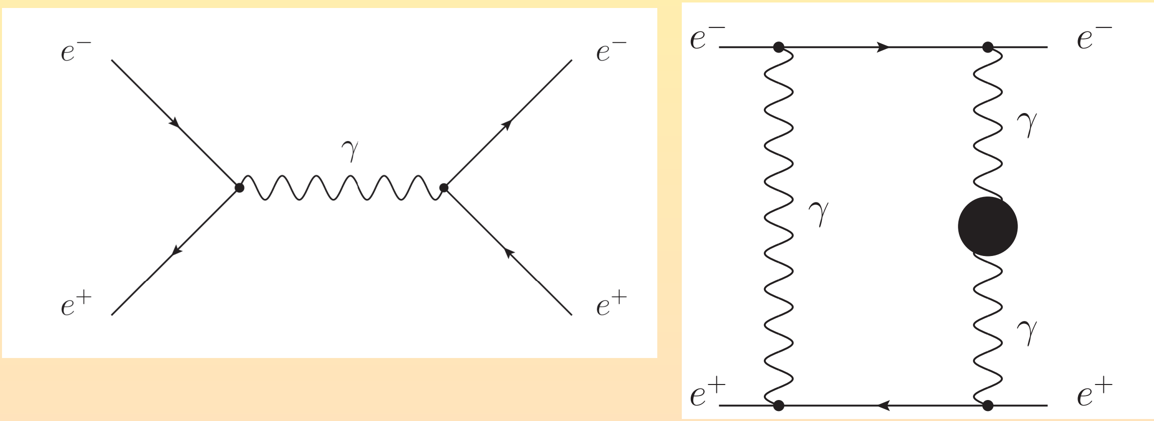
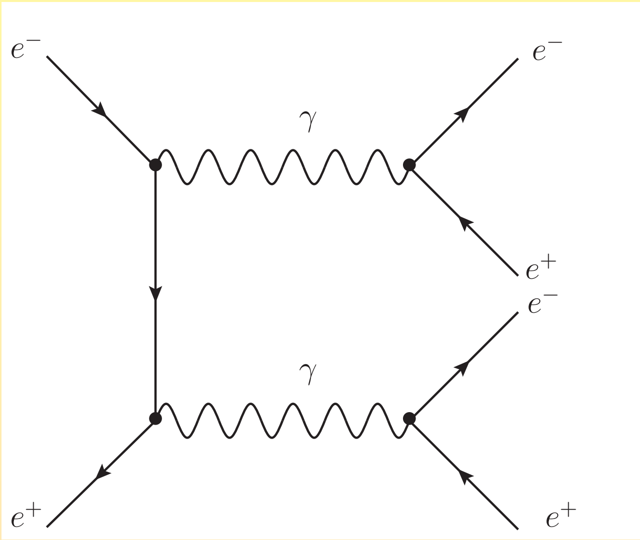
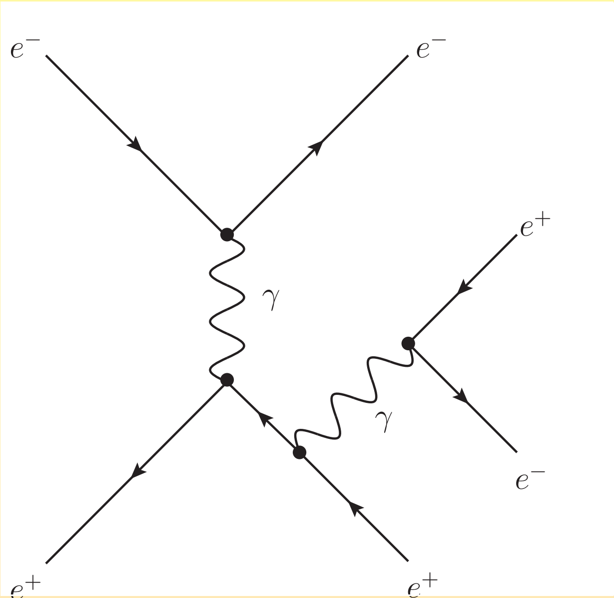
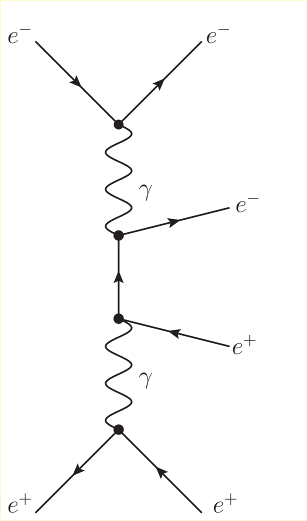


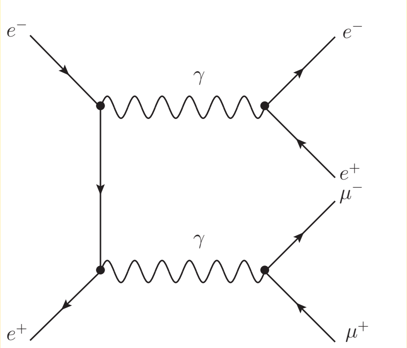
electron pairs

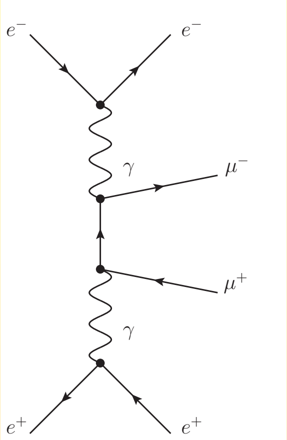


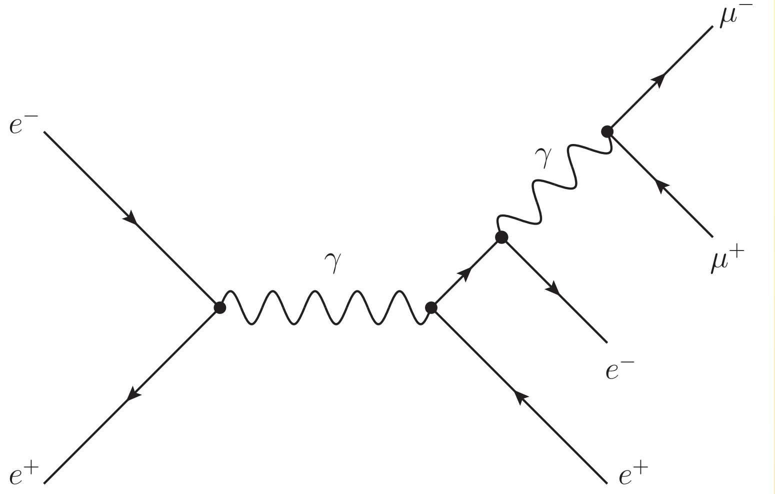




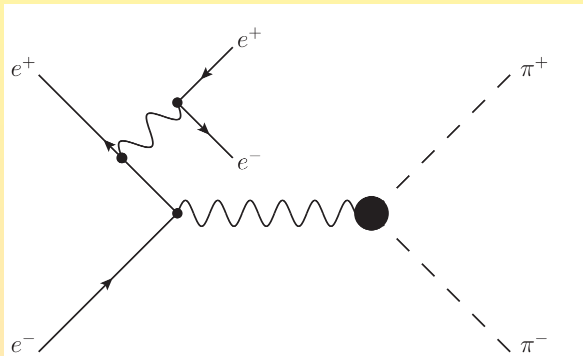
muon pairs

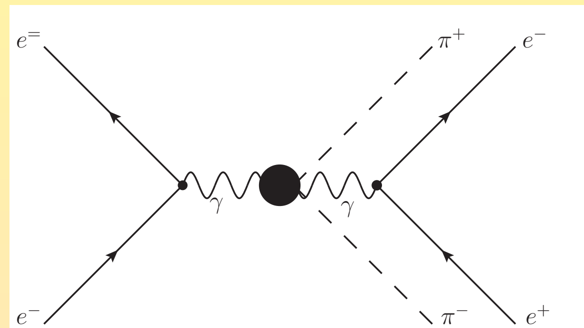

e^-
e^-
γ
e^+
μ^-
γ
e^+
μ^+

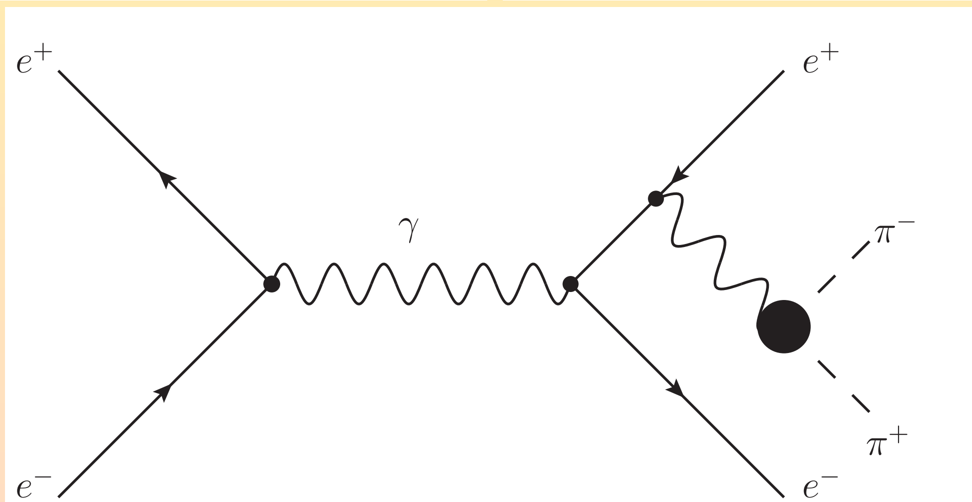

e^-
e^-
γ
μ^-
μ^+
γ
e^+
e^+


μ^-
e^-
γ
μ^+
γ
e^-
e^+
e^+

pion pairs

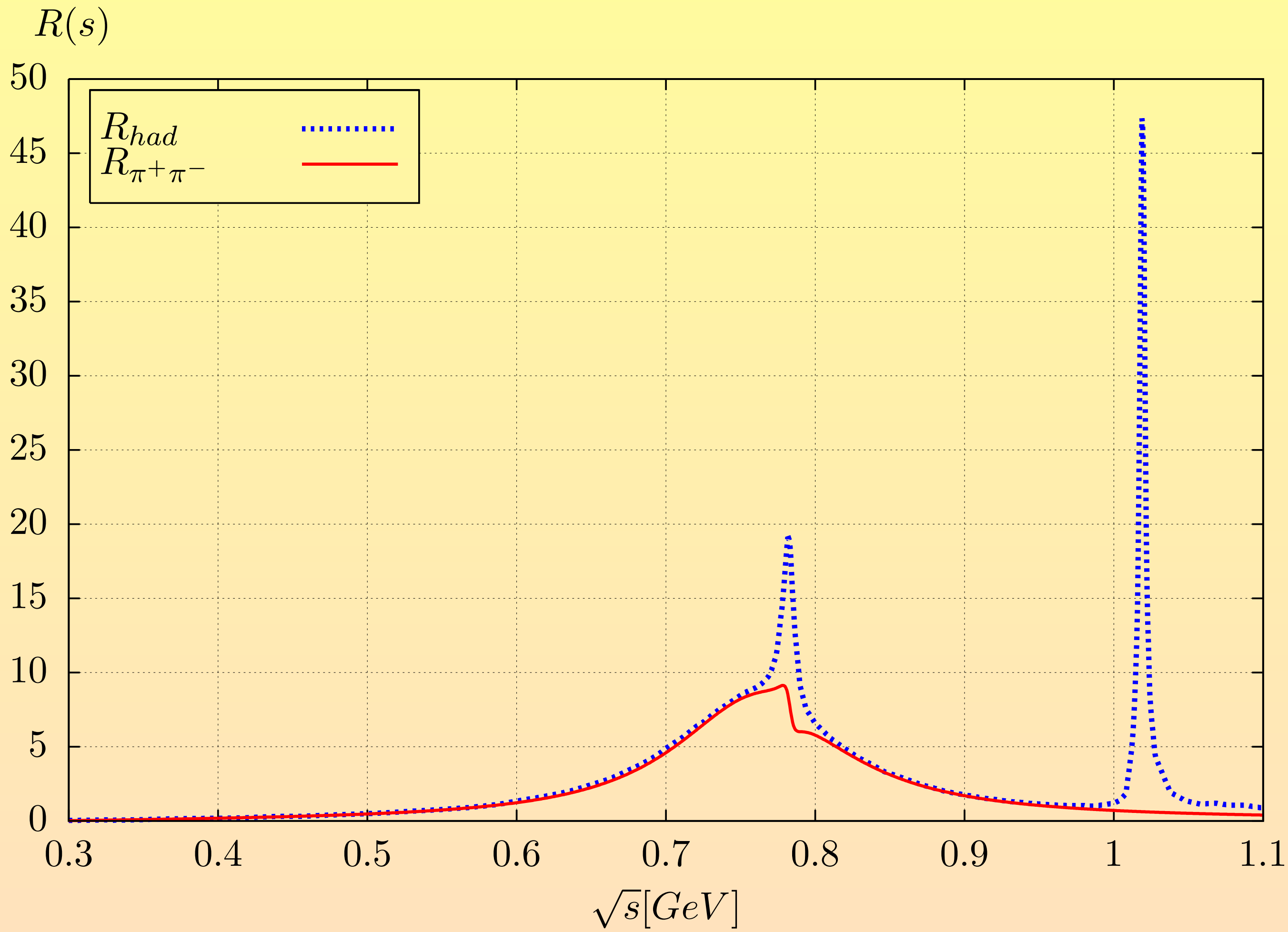






	$\sqrt{s}$	$\sigma^{\rm NNLO}_{\rm rest,res}$	$\sigma^{\rm NNLO}_{\rm rest,res'}$	σ_B
KLOE	1.020	[all n.r.] -0.04538	[n.r. without J/ψ(1S)] -0.0096	529.5
BES	3.097	[all n.r.] 228.08	[n.r. without J/ψ(1S)] -0.0258	14.75
BES	3.650	[all n.r.] -0.1907	[n.r. without ψ(2S)] -0.023668	123.94
BES	3.686	[all n.r.] -62.537	[n.r. without ψ(2S)] -0.0254	121.53
BaBar	10.56	[all n.r.] -0.0163	[n.r. without Υ(4S)] -0.01438	6.744
Belle	10.58	[all n.r.] 0.04393	[n.r. without Υ(4S)] -0.0137	6.331

resonance	$M_{\rm res}$ [GeV]	$\Gamma^{e^+e^-}_{\rm res}$ [keV]
J/ψ(1S)	3.096916	5.55
ψ(2S)	3.686093	2.33
Υ(1S)	9.46030	1.34
Υ(2S)	10.02326	0.612
Υ(3S)	10.3552	0.443
Υ(4S)	10.5794	0.272
Υ(5S)	10.865	0.31
Υ(6S)	11.019	0.13



$R(s)$
R_{had}
$R_{\pi^+\pi^-}$
50
45
40
35
30
25
20
15
10
5
0
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
$\sqrt{s}[GeV]$

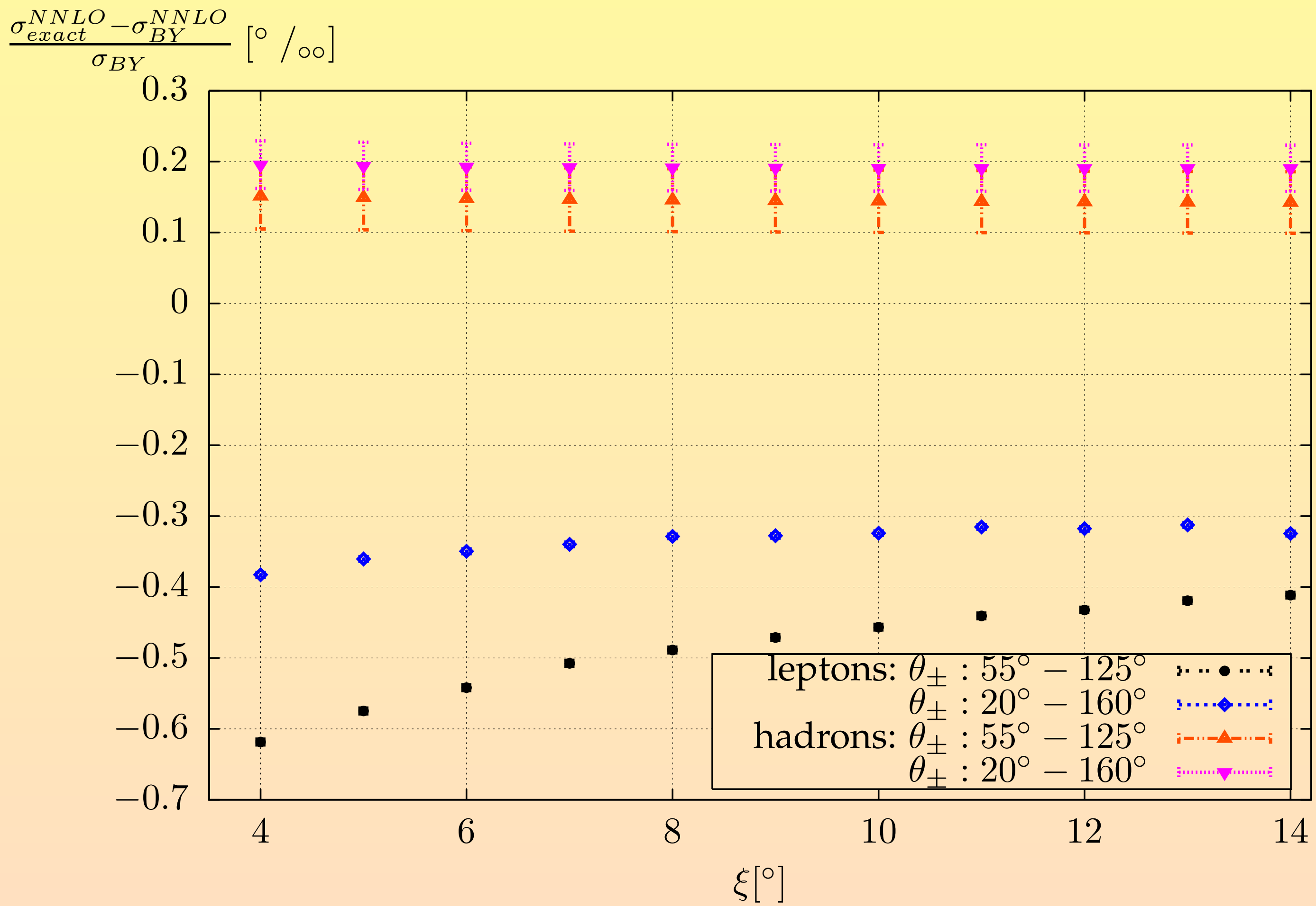
	$\sqrt{s}$		σ_{BY}	$S_{e^+e^-}$ [permille]	S_{lep} [permille]	S_{had} [permille]	S_{tot} [permille]
KLOE	1.020	NNLO		-3.935(4)	-4.472(4)	1.02(2)	-3.45(2)
		BABAYAGA(AT)NLO	455.71	-3.445(2)	-4.001(2)	0.876(5)	-3.126(5)
BES	3.097	NNLO		-2.246(8)	-2.771(8)	-	-
		BABAYAGA(AT)NLO	158.23	-2.019(3)	-2.548(3)	-	-
BES	3.650	NNLO		-1.469(9)	-1.913(9)	-1.3(1)	-3.2(1)
		BABAYAGA(AT)NLO	116.41	-1.521(4)	-1.971(4)	-1.071(4)	-3.042(5)
BES	3.686	NNLO		-1.435(8)	-1.873(8)	-	-
		BABAYAGA(AT)NLO	114.27	-1.502(4)	-1.947(4)	-	-
BaBar	10.56	NNLO		-1.48(2)	-2.17(2)	-1.69(8)	-3.86(8)
		BABAYAGA(AT)NLO	5.195	-1.40(1)	-2.09(1)	-1.49(1)	-3.58(2)
Belle	10.58	NNLO		-4.93(2)	-6.84(2)	-4.1(1)	-10.9(1)
		BABAYAGA(AT)NLO	5.501	-4.42(1)	-6.38(1)	-3.86(1)	-10.24(2)

The σ_{BY} is the cross section in nb from BABAYAGA(AT)N LO, and $S_x = \frac{\sigma_x^{\mathrm{NNLO}}}{\sigma_{\mathrm{BY}}}$ with $x = e^+e^-, lep, tot$, where tot stands for leptonic (lep) + hadronic corrections.

Conclusion:

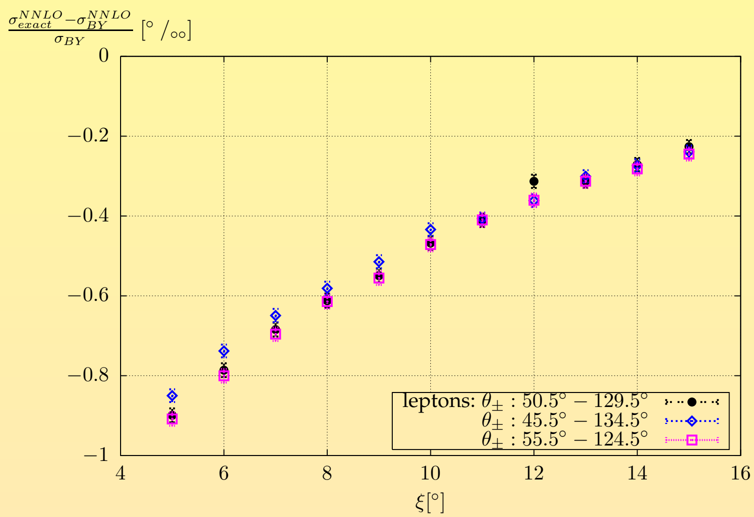
For realistic reference event selections we find agreement for the NNLO leptonic and hadronic corrections within 0.07% or better and conclude that they are well accounted for in the generator by comparison with the present experimental accuracy.

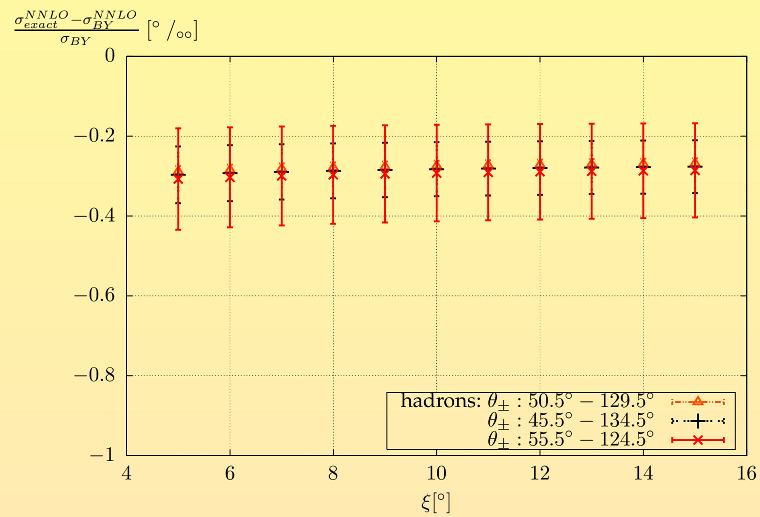
Kloe



$\frac{\sigma^{NNLO}_{exact} - \sigma^{NNLO}_{BY}}{\sigma_{BY}}$ [° /∘∘]
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
−0.4
−0.5
−0.6
−0.7
4
6
8
10
12
14
ξ[°]
leptons: $\theta_\pm$: 55° − 125°
$\theta_\pm$: 20° − 160°
hadrons: $\theta_\pm$: 55° − 125°
$\theta_\pm$: 20° − 160°

Belle





Contributions of leptons and hadrons to NNLO Bhabha process can be constructive (Belle) or destructive (Kloe), they also depends strongly for some colliders/detectors on kinematical cuts.

Conclusions

Miyamoto Musashi (1584 – 1645)

The Five Rings

The Broad Principles of Musashi's Strategy

- Do not think dishonestly.
- The Way is in training.
- Become aquainted with every art.
- Know the Ways of all professions.
- Distinguish between gain and loss in worldly matters.
- Develop intuitive judgement and understanding for everything.
- Perceive those things which cannot be seen.
- Pay attention even to trifles.
- Do nothing which is of no use.

Alles Gute Tord (Wszystkiego Najlepszego)