Precision EW physics at lepton colliders

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in collaboration with

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COST PARTICLEFACE WG MEETING

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WG3 objectives, tasks, milestones

- Clearly crosses in many points with WG1&WG2 - see milestones below
- Goals of WG3 worth to study from perspective of synergy of future hadron/lepton colliders (T. Lesiak talk)

WG3: Assessing the discovery potential of future high energy colliders	
Objectives	Assessment of the discovery potential of future high energy colliders based on the experimental results from the high-energy run of the LHC combined with progress in theoretical research.
Tasks	Study of EW gauge-boson production at fixed order in perturbation theory and resummation modelled through parton showers. The scattering of EW gauge-bosons at high energies is particularly important for further investigations of the EWSB at future colliders and the determination of the scale of new physics. Studies of heavy-quark properties and couplings in the SM and beyond as well as applications of effective field theories to jet observables at hadron colliders. As a particular strong point the research approach towards new physics phenomena is largely independent of particular model assumptions and driven by precision comparisons to SM predictions for well-defined signals. This maximizes the discovery potential.
Milestones	▪ Prospects for the physics of the Higgs boson, EW bosons, top quarks, and high-energy jets at future colliders with ultra-high multiplicities. ▪ Combined EW and QCD predictions at the highest energies. ▪ Resummation techniques for multi-scale processes at the highest energies.
Deliverables	▪ Publications in high-impact journals. ▪ Recommendations for the European strategy discussion of high-energy

50 years of the Z-boson theory (1967)

S. Weinberg

"A MODEL OF LEPTONS"

VOLUME 19, NUMBER 21 PHYSICAL REVIEW LETTERS 20 NOVEMBER 1967

and

$$\varphi_1 \equiv (\varphi^0 + \varphi^{0\dagger} - 2\lambda)/\sqrt{2} \qquad \varphi_2 \equiv (\varphi^0 - \varphi^{0\dagger})/i\sqrt{2}. \qquad (5)$$

The condition that φ_1 have zero vacuum expectation value to all orders of perturbation theory tells us that $\lambda^2 \cong M_1{}^2/2h$, and therefore the field φ_1 has mass M_1 while φ_2 and φ^- have mass zero. But we can easily see that the Goldstone bosons represented by φ_2 and φ^- have no physical coupling. The Lagrangian is gauge invariant, so we can perform a combined isospin and hypercharge gauge transformation which eliminates φ^- and φ_2 everywhere[6] without changing anything else. We will see that G_e is very small, and in any case M_1 might be very large,[7] so the φ_1 couplings will also be disregarded in the following.

The effect of all this is just to replace φ everywhere by its vacuum expectation value

$$\langle\varphi\rangle = \lambda\binom{1}{0}. \qquad (6)$$

The first four terms in $\mathcal{L}$ remain intact, while the rest of the Lagrangian becomes

$$-\tfrac{1}{8}\lambda^2 g^2[(A_\mu{}^1)^2 + (A_\mu{}^2)^2] - \tfrac{1}{8}\lambda^2(gA_\mu{}^3 + g'B_\mu)^2 - \lambda G_e \bar{e}e. \qquad (7)$$

We see immediately that the electron mass is λG_e. The charged spin-1 field is

$$W_\mu \equiv 2^{-1/2}(A_\mu{}^1 + iA_\mu{}^2) \qquad (8)$$

and has mass

$$M_W = \tfrac{1}{2}\lambda g. \qquad (9)$$

The neutral spin-1 fields of definite mass are

$$Z_\mu = (g^2 + g'^2)^{-1/2}(gA_\mu{}^3 + g'B_\mu), \qquad (10)$$

$$A_\mu = (g^2 + g'^2)^{-1/2}(-g'A_\mu{}^3 + gB_\mu). \qquad (11)$$

Their masses are

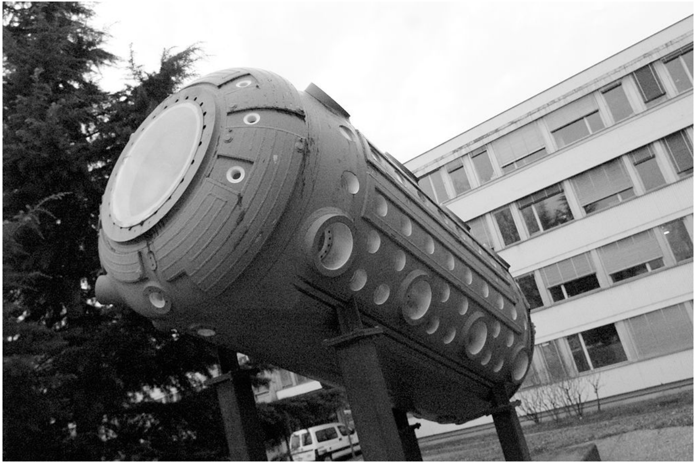
$$M_Z = \tfrac{1}{2}\lambda(g^2 + g'^2)^{1/2}, \qquad (12)$$

$$M_A = 0, \qquad (13)$$

so A_μ is to be identified as the photon field. The interaction between leptons and spin-1 mesons is

$$\frac{ig}{2\sqrt{2}}\bar{e}\gamma^\mu(1+\gamma_5)\nu W_\mu + \text{H.c.} + \frac{igg'}{(g^2+g'^2)^{1/2}}\bar{e}\gamma^\mu e A_\mu + \frac{i(g^2+g'^2)^{1/2}}{4}\left[\left(\frac{3g'^2-g^2}{g'^2+g^2}\right)\bar{e}\gamma^\mu e - \bar{e}\gamma^\mu\gamma_5 e + \bar{\nu}\gamma^\mu(1+\gamma_5)\nu\right]Z_\mu. \qquad (14)$$

And, exactly 45 years of the Z-boson discovery (1973)



Gargamelle

Rich physics

Erler, Freitas, PDG'17

Presently:

Very good agreement

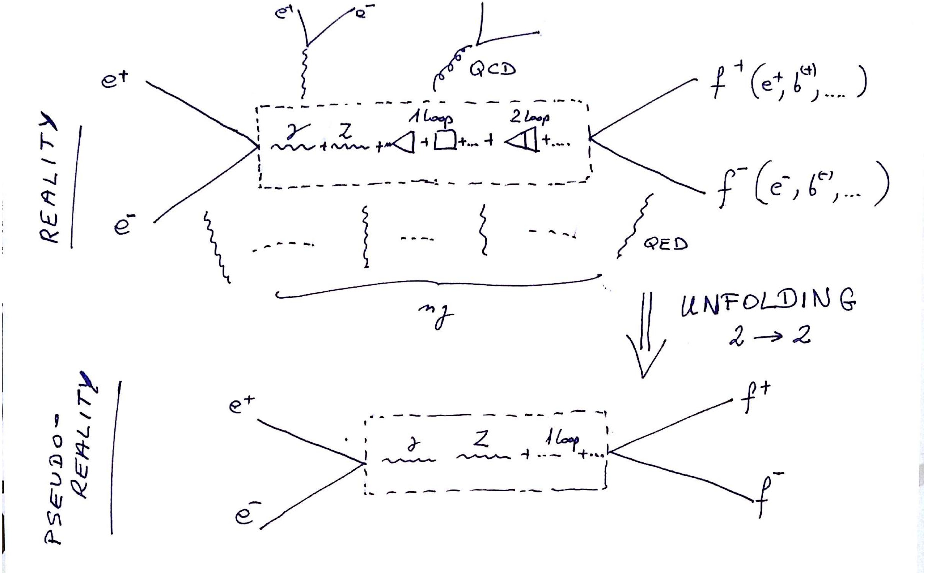
theory — experiment

over large number of EWPOs

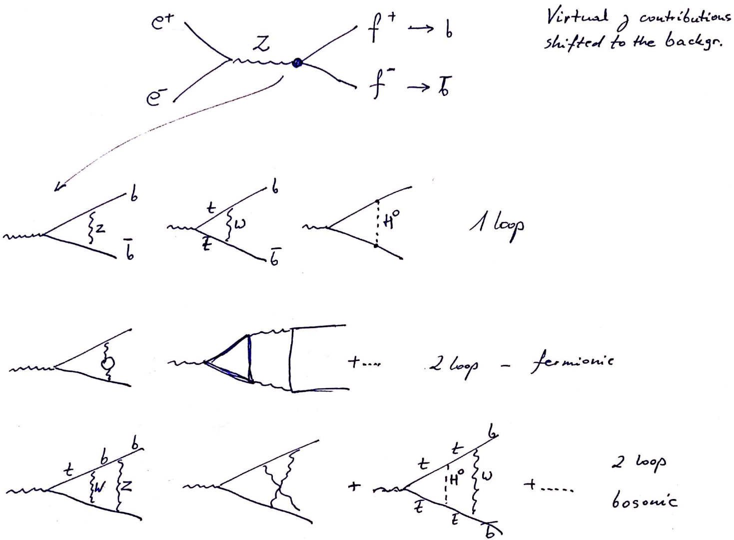
Table 10.5: Principal Z pole observables and their SM predictions (*cf.* Table 10.4). The first $\bar{s}_\ell^2$ is the effective weak mixing angle extracted from the hadronic charge asymmetry, the second is the combined value from the Tevatron [164–166], and the third from the LHC [170–172]. The values of A_e are (i) from A_{LR} for hadronic final states [159]; (ii) from A_{LR} for leptonic final states and from polarized Bhabba scattering [161]; and (iii) from the angular distribution of the τ polarization at LEP 1. The A_τ values are from SLD and the total τ polarization, respectively.

Quantity	Value	Standard Model	Pull
M_Z [GeV]	91.1876 ± 0.0021	91.1880 ± 0.0020	-0.2
Γ_Z [GeV]	2.4952 ± 0.0023	2.4943 ± 0.0008	0.4
$\Gamma(\text{had})$ [GeV]	1.7444 ± 0.0020	1.7420 ± 0.0008	—
$\Gamma(\text{inv})$ [MeV]	499.0 ± 1.5	501.66 ± 0.05	—
$\Gamma(\ell^+\ell^-)$ [MeV]	83.984 ± 0.086	83.995 ± 0.010	—
σ_{had} [nb]	41.541 ± 0.037	41.484 ± 0.008	1.5
R_e	20.804 ± 0.050	20.734 ± 0.010	1.4
R_μ	20.785 ± 0.033	20.734 ± 0.010	1.6
R_τ	20.764 ± 0.045	20.779 ± 0.010	-0.3
R_b	0.21629 ± 0.00066	0.21579 ± 0.00003	0.8
R_c	0.1721 ± 0.0030	0.17221 ± 0.00003	0.0
$A_{FB}^{(0,e)}$	0.0145 ± 0.0025	0.01622 ± 0.00009	-0.7
$A_{FB}^{(0,\mu)}$	0.0169 ± 0.0013		0.5
$A_{FB}^{(0,\tau)}$	0.0188 ± 0.0017		1.5
$A_{FB}^{(0,b)}$	0.0992 ± 0.0016	0.1031 ± 0.0003	-2.4
$A_{FB}^{(0,c)}$	0.0707 ± 0.0035	0.0736 ± 0.0002	-0.8
$A_{FB}^{(0,s)}$	0.0976 ± 0.0114	0.1032 ± 0.0003	-0.5
$\bar{s}_\ell^2$	0.2324 ± 0.0012	0.23152 ± 0.00005	0.7
	0.23185 ± 0.00035		0.9
	0.23105 ± 0.00087		-0.5
A_e	0.15138 ± 0.00216	0.1470 ± 0.0004	2.0
	0.1544 ± 0.0060		1.2
	0.1498 ± 0.0049		0.6
A_μ	0.142 ± 0.015		-0.3
A_τ	0.136 ± 0.015		-0.7
	0.1439 ± 0.0043		-0.7
A_b	0.923 ± 0.020	0.9347	-0.6
A_c	0.670 ± 0.027	0.6678 ± 0.0002	0.1
A_s	0.895 ± 0.091	0.9356	-0.4

Rough scheme for extracting the $Z\bar{f}f$ vertex and EW corrections (1)



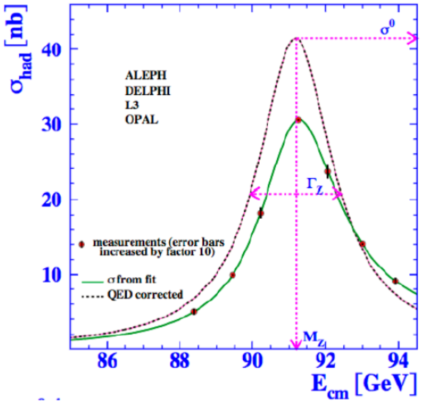
Rough scheme for extracting the $Z\bar{f}f$ vertex and EW corrections (2)



LEP sensitive to SM radiative corrections

Altogether $5 \cdot 10^6$ Z-boson decays.

- Cross section : Z mass and width



- ~30% QED corrections (ISR)

σ_0 seen from experiments (needs knowing QED)

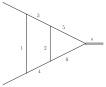
EWPOs (electroweak pseudo-observables):

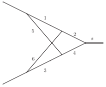
$$\sigma^{real}_{peak} \longrightarrow \begin{cases} \sigma_0 \equiv \sigma^{eff.,Born}_{peak} \\ M_Z, \Gamma_Z, \Gamma_{partial} \\ A^{eff.,Born}_{FB,peak}, A^{eff.,Born}_{LR,peak} \\ R_b, R_\ell \end{cases}$$

- Not got for free! Unfolding of QED Improvements needed for basic LEP programs: KKMC, ZFITTER,...

EWPOs & Form Factors

$$V_\mu^{Zb\bar{b}} = \gamma_\mu[v_b(s) - a_b(s)\gamma_5] = \cdots + \overbrace{\underbrace{D_1 + D_2}_{\text{planar,non}-\text{planar}}}^{\text{fermionic,bosonic}} + \cdots$$





Note approximate factorization of weak couplings

$$A_{F-B} = \frac{\left[\int_0^1 d\cos\theta - \int_{-1}^0 d\cos\theta\right]\frac{d\sigma}{d\cos\theta}}{\sigma_T} \sim \overbrace{\frac{2a_e v_e}{a_e^2+v_b^2}}^{A_e}\ \overbrace{\frac{\mathbf{2a_b v_b}}{\mathbf{a_b^2+v_b^2}}}^{A_b} + \text{corrections} \leftarrow (\text{Tord})$$

$$A_{\mathrm{b}} = \frac{2\Re e\frac{v_b}{a_b}}{1+\left(\Re e\frac{v_b}{a_b}\right)^2} = \frac{1-4|Q_b|\sin^2\theta_{\mathrm{eff}}^{\mathrm{b}}}{1-4|Q_b|\sin^2\theta_{\mathrm{eff}}^{\mathrm{b}} + 8Q_b^2(\sin^2\theta_{\mathrm{eff}}^{\mathrm{b}})^2}, \quad \sin^2\theta_{\mathrm{eff}}^{\mathrm{b}} \longrightarrow F\left(\Re e\frac{v_b}{a_b}\right)$$

EW SM theory at loops, an example ($\Delta_{ef} \neq 0$)

$$\left\{ \begin{array}{l} \Gamma_Z, \Gamma_{partial} \\ \\ A^{eff.,Born}_{FB,peak}, A^{eff.,Born}_{LR,peak} \\ \\ R_b, R_\ell, \ldots \end{array} \right. \longrightarrow \left\{ \begin{array}{l} v^{eff}_{\ell,\nu,u,d,b} \\ \\ a^{eff}_{\ell,\nu,u,d,b} \\ \\ \sin^2\theta^{\rm b}_{\rm eff}, \sin^2\theta^{lept}_{\rm eff} \end{array} \right.$$

e.g. : improvements needed for subtle corrections $\Delta_{1,2}$ (e.g. boxes, **2L-boxes**)

$$A^{eff.,Born}_{FB,peak} = \frac{2\Re e\left[\frac{v_e a_e^*}{|a_e|^2}\right] 2\Re e\left[\frac{v_f a_f^*}{|a_f|^2}\right]}{\left(1+\frac{|v_e|^2}{|a_e|^2}\right)\left(1+\frac{|v_f|^2}{|a_f|^2}\right)} + \Delta_1 - \Delta_2 \simeq \frac{3}{4} A_e A_f,$$

$$\Delta_1 = 2\Re e\left[\Delta_{ef}\right], \; \Delta_2 = |\Delta_{ef}|^2 + 2\Re e\left[\frac{v_e a_e^*}{|a_e|^2}\frac{v_f a_f^*}{|a_f|^2}\Delta^*_{ef}\right],$$

$$\Delta_{ef} = 16|Q_e Q_f| s_W^4 (\kappa_{ef} - \kappa_e \kappa_f)$$

A. Blondel: FCC-ee experimental demands to theory - "Giga-Z"

observable	Physics	Present precision		FCC-ee stat Syst Precision	FCC-ee key	Challenge
M_Z MeV/c2	Input	91187.5 ±2.1	Z Line shape scan	0.005 MeV <±0.1 MeV	E_cal	QED corrections
Γ_Z MeV/c2	$\Delta\rho$ (T) (no $\Delta\alpha$!)	2495.2 ±2.3	Z Line shape scan	0.008 MeV <±0.1 MeV	E_cal	QED corrections
$R_l = \frac{\Gamma_h}{\Gamma_l}$	α_s, δ_b	20.767 (25)	Z Peak	0.0001 (2-20)	Statistics	QED corrections
N_ν	Unitarity of PMNS, sterile ν's	2.984 ±0.008	Z Peak Z+γ(161 GeV)	0.00008 (40) 0.001	->lumi meast Statistics	QED corrections to Bhabha scat.
R_b	δ_b	0.21629 (66)	Z Peak	0.000003 (20-60)	Statistics, small IP	Hem. corr, gluon split. m_b
A_{LR}	$\Delta\rho$, ε_3, $\Delta\alpha$ (T, S)	$\sin^2\theta_w^{eff}$ 0.23098(26)	Z peak, Long. polarized	$\sin^2\theta_w^{eff}$ ±0.000006	4 bunch scheme	Design experiment
A_{FB}^{lept}	$\Delta\rho$, ε_3, $\Delta\alpha$ (T, S)	$\sin^2\theta_w^{eff}$ 0.23099(53)		$\sin^2\theta_w^{eff}$ ±0.000006	E_cal & Statistics	
M_W MeV/c2	$\Delta\rho$, ε_3, ε_2, $\Delta\alpha$ (T, S, U)	80385 ± 15	Threshold (161 GeV)	0.3 MeV <0.5 MeV	E_cal & Statistics	QED corections
m_{top} MeV/c2	Input	173200 ± 900	Threshold scan	~10 MeV	E_cal & Statistics	Theory limit at 50 MeV?

Main issue

A. Blondel: A BIG QUESTION

Can theory in 2040 ($\simeq$data taking) comply with the level of anticipated experimental accuracy?

To answer, in this talk I will discuss:

- Case of Γ_Z intrinsic accuracy (preliminary results) vs. exp. demand **[0.1 MeV]**;

Current uncertainities, Ayres: 1604.00406

	Experiment	Theory error	Main source
M_W	80.385 ± 0.015 MeV	4 MeV	$\alpha^3,\ \alpha^2\alpha_s$
Γ_Z	2495.2 ± 2.3 MeV	0.5 MeV	$\alpha_{bos}^2,\ \alpha^3,\ \alpha^2\alpha_s,\ \alpha\alpha_s^2$
σ_{had}^0	41540 ± 37 pb	6 pb	$\alpha_{bos}^2,\ \alpha^3,\ \alpha^2\alpha_s$
$R_b \equiv \Gamma_Z^b/\Gamma_Z^{had}$	0.21629 ± 0.00066	0.00015	$\alpha_{bos}^2,\ \alpha^3,\ \alpha^2\alpha_s$
$\sin^2\theta_{eff}^{\ell}$	0.23153 ± 0.00016	4.5×10^{-5}	$\alpha^3,\ \alpha^2\alpha_s$

This talk: very preliminary results will be shown and discussed

Future projections, Ayres: 1604.00406

	Measurement error			Intrinsic theory	
	ILC	CEPC	FCC-ee	Current	Future†
M_{W} [MeV]	3–4	3	1	4	1
Γ_{Z} [MeV]	0.8	0.5	**0.1**	**0.5**	**0.2**
R_b $[10^{-5}]$	14	17	6	15	7
$\sin^2\theta^{\ell}_{\mathrm{eff}}$	1	2.3	**0.6**	4.5	**1.5**

Table: Projected experimental and theoretical uncertainties for some electroweak precision pseudo-observables.

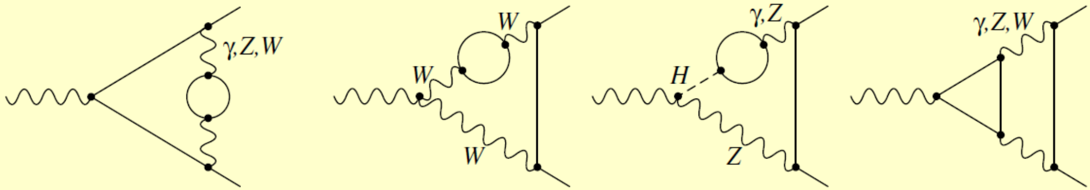
† Based on estimations for: $\mathcal{O}(\alpha^2_{bos})$,$\mathcal{O}(\alpha\alpha_s^2)$, $\mathcal{O}(N_f\alpha^2\alpha_s)$, $\mathcal{O}(N_f^2\alpha^3)$

Our main goal ahead:

Present theoretical accuracy is not enough

Vocabulary

Known corrections ($\Delta\rho$, $\sin^2\theta^{\rm f}_{\rm eff}, g_V, g_A$) comes from fermionic part (fermions loops)



and rest constitute so-called bosonic corrections.

Published results on EWPOs in the SM @NNLO

Complete corrections $\Delta r, \sin^2\theta^l_{\text{eff}}$:
- Freitas, Hollik, Walter, Weiglein: '00
- Awramik,Czakon: '02,Onishchenko,Veretin: '02
- Awramik,Czakon,Freitas,Weiglein: '04
- Awramik,Czakon,Freitas: '06
- Hollik,Meier,Uccirati: '05,'07
- Degrassi,Gambino,Giardino: '14

Fermionic corrections $\sin^2\theta^b_{\text{eff}}, a_f, v_f$:
- Awramik,Czakon,Freitas,Kniehl: '09
- Czarnecki,Kühn: '96
- Harlander,Seidensticker,Steinhauser: '98
- Freitas: '13,'14

Bosonic corrections $\sin^2\theta^{\text{b}}_{\text{eff}}$: Dubovyk, Freitas, JG, Riemann, Usovitsch '16

This talk: Bosonic corrections a_f, v_f : Dubovyk, Freitas, JG, Riemann, Usovitsch '18

What we need: error estimations, Ayres: 1604.00406

- Theory error estimate is not well defined, ideally $\Delta_{\mathsf{th}} \ll \Delta_{\mathsf{exp}}$
- Common methods:
  - Count prefactors (α, N_c, N_f, ...)
  - Extrapolation of perturbative series
  - Renormalization scale dependence
  - Renormalization scheme dependence
- Also parametric error from external inputs (m_{t}, m_b, α_{s}, $\Delta\alpha_{\mathsf{had}}$, ...)

An example: Intrinsic theory error estimation for Γ_Z, Ayres: 1604.00406

1. Geometric series

$$\delta_1 : \mathcal{O}(\alpha^3) - \mathcal{O}(\alpha_t^3) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)}\mathcal{O}(\alpha^2) \sim 0.26 \text{ MeV}$$

$$\delta_2 : \mathcal{O}(\alpha^2\alpha_s) - \mathcal{O}(\alpha_t^2\alpha_s) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)}\mathcal{O}(\alpha\alpha_s) \sim 0.3 \text{ MeV}$$

$$\delta_3 : \mathcal{O}(\alpha\alpha_s^2) - \mathcal{O}(\alpha_t\alpha_s^2) \sim \frac{\mathcal{O}(\alpha\alpha_s) - \mathcal{O}(\alpha_t\alpha_s)}{\mathcal{O}(\alpha)}\mathcal{O}(\alpha\alpha_s) \sim 0.23 \text{ MeV}$$

$$\delta_4 : \mathcal{O}(\alpha\alpha_s^3) - \mathcal{O}(\alpha_t\alpha_s^3) \sim \frac{\mathcal{O}(\alpha\alpha_s) - \mathcal{O}(\alpha_t\alpha_s)}{\mathcal{O}(\alpha)}\mathcal{O}(\alpha\alpha_s^2) \sim 0.035 \text{ MeV}$$

$$\delta_5 : \mathcal{O}(\alpha_{bos}^2) \sim \mathcal{O}(\alpha_{bos})^2 \sim \mathbf{0.1} \text{ MeV } \underline{\text{[Now we know it!]}}$$

Total: $\delta\Gamma_Z = \sqrt{\sum_{i=1}^{5} \delta_i^2} \sim \mathbf{0.5}$ **MeV**

New results for completing NNLO

Input parameters:

Parameter	Value	Parameter	Value
$M_{\rm Z}$	91.1876 GeV	$m_{\rm b}^{\overline{\rm MS}}$	4.20 GeV
$\Gamma_{\rm Z}$	2.4952 GeV	$m_{\rm c}^{\overline{\rm MS}}$	1.275 GeV
$M_{\rm W}$	80.385 GeV	m_{τ}	1.777 GeV
$\Gamma_{\rm W}$	2.085 GeV	$\Delta\alpha$	0.05900
$M_{\rm H}$	125.1 GeV	$\alpha_{\rm s}(M_{\rm Z})$	0.1184
$m_{\rm t}$	173.2 GeV	G_{μ}	1.16638×10^{-5} GeV^{-2}

The cherry on the 2-loops EWPOs cake: results for $\mathcal{O}(\alpha_{\rm bos}^2)$ **[preliminary]**[*]

Γ_i [MeV]	$\Gamma_e, \Gamma_\mu, \Gamma_\tau$	$\Gamma_{\nu_e}, \Gamma_{\nu_\mu}, \Gamma_{\nu_\tau}$	Γ_d, Γ_s	Γ_u, Γ_c	Γ_b	Γ_Z
$\mathcal{O}(\alpha)$	2.273	6.174	9.717	5.799	3.857	60.22
$\mathcal{O}(\alpha\alpha_{\rm s})$	0.288	0.458	1.276	1.156	2.006	9.11
$\mathcal{O}(N_f^2\alpha^2)$	0.244	0.416	0.698	0.528	0.694	5.13
$\mathcal{O}(N_f\alpha^2)$	0.120	0.185	0.493	0.494	0.144	3.04
$\mathcal{O}(\alpha_{\rm bos}^2)$	**0.017**	**0.019**	**0.058**	**0.057**	**0.167**	**0.505**
$\mathcal{O}(\alpha_{\rm t}\alpha_{\rm s}^2,\ \alpha_{\rm t}\alpha_{\rm s}^3,\ \alpha_{\rm t}^2\alpha_{\rm s},\ \alpha_{\rm t}^3)$	0.038	0.059	0.191	0.170	**0.190**	1.20

1. Fun fact of the day: so far all contributions positive!
2. 2016, estimation, bosonic NNLO $\sim 0 \pm 0.1$ MeV

   2018, exact result: 0.505 MeV

* I. Dubovyk, A. Freitas, JG, T. Riemann, J. Usovitsch

Having this knowledge: **genuine** 3-loop vertex calculations are obligatory!

1. Geometric series

$$\mathcal{O}(\alpha^3) - \mathcal{O}(\alpha_t^3) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha^2) \sim \text{~~0.26 MeV~~}$$

$$\mathcal{O}(\alpha^2\alpha_s) - \mathcal{O}(\alpha_t^2\alpha_s) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha\alpha_s) \text{~~$\sim$ 0.3 MeV~~}$$

$$\mathcal{O}(\alpha\alpha_s^2) - \mathcal{O}(\alpha_t\alpha_s^2) \sim \frac{\mathcal{O}(\alpha\alpha_s) - \mathcal{O}(\alpha_t\alpha_s)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha\alpha_s) \sim \text{~~0.23 MeV~~}$$

$$\mathcal{O}(\alpha\alpha_s^3) - \mathcal{O}(\alpha_t\alpha_s^3) \sim \frac{\mathcal{O}(\alpha\alpha_s) - \mathcal{O}(\alpha_t\alpha_s)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha\alpha_s^2) \sim \text{~~0.035 MeV~~}$$

$$\mathcal{O}(\alpha_{bos}^2) \sim \mathcal{O}(\alpha_{bos})^2 \sim \text{~~\textbf{0.1 MeV} $\to$ [\textbf{0.51} MeV]~~}$$

1. FCC-ee$^{exper.\ error}(\Gamma_Z) \sim 0.1$ MeV
2. FCC-ee$^{theor.\ error}(\Gamma_Z) <$ FCC-ee$^{exper.\ error}(\Gamma_Z)$???

Answering Alain Blondel's Big Question

- Calculating N^3LO with 10% accuracy (two digits), we can replace 2016 intrinsic error estimation $\delta\Gamma_Z = \sqrt{\sum_{i=1}^{5} \delta_i^2} \sim 0.5$ MeV by $\delta\Gamma_Z = \sqrt{\sum_{i=1}^{5} (\delta_i/10)^2} \sim 0.05$ MeV.

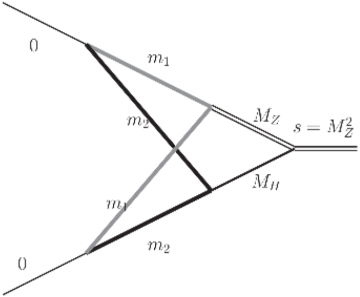
- The requirement of FCC-ee$^{exper.\ error}(\Gamma_Z) \sim 0.1$ MeV can be met and the condition

$$\delta[\mathrm{FCCee}^{\mathrm{theor.}}(\Gamma_\mathrm{Z})] \sim 0.05\ \mathrm{MeV} < \delta[\mathrm{FCCee}^{exper.}(\Gamma_Z)] \sim 0.1\ \mathrm{MeV}$$

will be fullfilled.

QCD 4-loops(!)

2-loops $\longrightarrow$ 3-loops



$$m_1 = M_t, m_2 = M_W$$

The integrals contain up to three dimensionless parameters

$$\left\{ \frac{M_H^2}{M_Z^2}, \frac{M_W^2}{M_Z^2}, \frac{m_t^2}{M_Z^2}, \frac{(M_Z + i\varepsilon)^2}{M_Z^2} \right\}$$

2-loops $\longrightarrow$ 3-loops

- The standard model prediction for the effective weak mixing angle can be written as

$$\sin^2\theta^{\mathrm{b}}_{\mathrm{eff}} = \left(1 - \frac{M_W^2}{M_Z^2}\right)(1 + \Delta\kappa_{\mathrm{b}})$$

- The bosonic electroweak two-loop corrections amount to

$$\boxed{\Delta\kappa_{\mathrm{b}}^{(\alpha^2,\mathrm{b}os)} = -0.9855 \times 10^{-4}}$$

DFGRU, Phys.Lett. B762 (2016) 184

Collection of radiative corrections: Full stabilization at 10^{-4}!

Order	Value $[10^{-4}]$	Order	Value $[10^{-4}]$
α	468.945	$\alpha_t^2\alpha_s$	1.362
$\alpha\alpha_s$	-42.655	α_t^3	0.123
$\alpha^2_{\rm ferm}$	3.866	$\alpha_t\alpha_s^2$	-7.074
$\alpha^2_{\rm bos}$	**-0.9855**	$\alpha_t\alpha_s^3$	-1.196

$\pm 0.001 \xrightarrow{!}$

Table: *Comparison of different orders of radiative corrections to* $\Delta\kappa_{\rm b}$.

Input Parameters: M_Z, Γ_Z, M_W, Γ_W, M_H, m_t, α_s *and* $\Delta\alpha$

- one-loop contributions [Akhundov, Bardin, Riemann, 1986] [Beenakker, Hollik, 1988]
- two-loop fermionic contributions [Awramik, Czakon, Freitas, Kniehl, 2009]
- two-loop bosonic contributions [Dubovyk, Freitas, JG, Riemann, Usovitsch, 2016]

Partial higher-order corrections

$\mathcal{O}(\alpha_{\rm t}\alpha_{\rm s}^2)$ Avdeev: 1994,Chetyrkin: 1995

$\mathcal{O}(\alpha_{\rm t}\alpha_{\rm s}^3)$ Schroder: 2005,Chetyrkin: 2006,Boughezal: 2006

$\mathcal{O}(\alpha^2\alpha_{\rm t})$ and $\mathcal{O}(\alpha_{\rm t}^3)$ vanderBij: 2000,Faisst: 2003

Mellin-Barnes and Sector Decomposition methods are very much complementary

- MB works well for hard threshold, on-shell cases, not many internal masses (more IR); SD more useful for integrals with many internal masses
  $\longrightarrow$ talks by Evgen, Johann and Sophia;
  $\longrightarrow$ JG, Kajda, Riemann, Yundin, EPJC'11; JG in PoS-LL2016 & DFGRU in PLB'16.

10^{-8} accuracy achieved for **any** self-energy and vertex Feynman integral

with one of the methods.

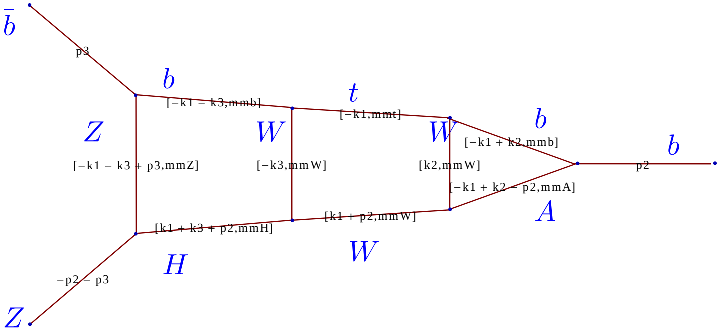
3-loops. Basic bookkeeping

$Z \to b\bar{b}$			
Number of topologies	1 loop	2 loops	3 loops
	1	14$\to^{(\mathbf{A})}$ 7 $\to^{(\mathbf{B})}$ **5**	211 $\to^{(\mathbf{A})}$ 84 $\to^{(\mathbf{B})}$ **50**
Number of diagrams	15	2383$\to^{(\mathbf{A,B})}$ **1114**	490387 $\to^{(\mathbf{A,B})}$**120187**
Fermionic loops	0	150	**17580**
Bosonic loops	15	**964**	102607
Planar diagrams	1T/15D	4T/981D	35T/84059D
Non-planar diagrams	0	1T/133D	15T/36128D

Table: Some statistical overview for $Z \to b\bar{b}$ multiloop studies. At 3 loops there are in total almost half a million of diagrams present. After basic refinements (A) and (B) about 10^5 genuine 3-loop vertex diagrams remain. In (A) tadpoles and products of lower loops are excluded, in (B) symmetries of topologies are taken into account.

A complete zoo of heavy particles m_t, m_W, m_Z, m_H @NNNLO level

MB: ϵ^0**[8-dim]**, $1/\epsilon$**[7-dim]**; SD: ϵ^0**[8-dim]** , $1/\epsilon$**[7-dim]**;



At 2-loops up to three dimensionless parameters (all 4 at 3-loops):

$$\left\{ \frac{M_H^2}{M_Z^2}, \frac{M_W^2}{M_Z^2}, \frac{m_t^2}{M_Z^2}, \frac{(M_Z + i\varepsilon)^2}{M_Z^2} \right\}$$

Conclusions on Z-lineshape and EWPOs for next years - theory

- Strong demand from FCC-ee to the theory on precision;
- We have to guarantee precise chain:
  $\sigma^{real} \rightarrow \text{pseudoobservables} \rightarrow> $ 2-loops in SM
- NNLO practically done, we need to go beyond:
  $\mathcal{O}(\alpha\alpha_s^2), \mathcal{O}(N_f\alpha^2\alpha_s), \mathcal{O}(N_f^2\alpha^3)$;
- 1. **We know how** to do it;
  2. and **we have appropriate tools;**
- To be on the safe side, we would like to have **at least 2 independent calculations**;
- Still, a lot work is ahead, for success and efficiency, **we need steady progress in numerical and also (semi)analytical approaches** in multiloop calculations

Backup slides

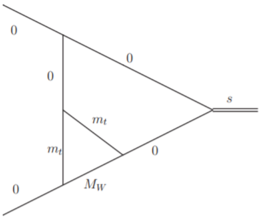
Sector decomposition

FIESTA 3 [A.V.Smirnov, 2014], SecDec 3 [Borowka, et. al., 2015] and pySecDec [Borowka, et. al., 2017]

Mellin-Barnes integral approach

- With AMBRE 2 [Gluza, et. al., 2011] (AMBRE 3 [Dubovyk, et. al., 2015]) we derive Mellin-Barnes representations for planar (non-planar) topologies. One may use PlanarityTest [Bielas, et. al, 2013] for automatic identification.
- Expansion in terms of $\epsilon = (4 - D)/2$ is done with MB [Czakon, 2006], MBresolve [A. Smirnov, V. Smirnov, 2009], barnesroutines (D. Kosower).
- For the numerical treatment of massive Mellin-Barnes integrals with Minkowskian regions, the package MBnumerics is being developed since 2015.

soft7 ϵ^0:[MB - 3 dim] [SD - 5 dim], ϵ^{-1}:[MB - 2 dim] [SD - 4 dim], ϵ^{-2}:[MB - 1 dim] [SD - 3 dim]



MB	$0.0602664865576999\,\epsilon^{-2}$	
SD - 90 Mio	$0.06026648655\,\epsilon^{-2}$	
MB	$(-0.03151248903$	$+0.18933275142i)\,\epsilon^{-1}$
SD - 90 Mio	$(-0.0315124816$	$+0.18933271696i)\,\epsilon^{-1}$
MB 1	$(-0.228231867511$	$-0.088247945691i) + \mathcal{O}(\epsilon)$
MB 2	$(-0.228231867551$	$-0.088247945739i) + \mathcal{O}(\epsilon)$
SD - 90 Mio	$(-0.22822653$	$-0.08824596i) + \mathcal{O}(\epsilon)$
SD - 15 Mio	$(-0.228162$	$-0.088209i) + \mathcal{O}(\epsilon)$

Intermezzo: $1997 \longrightarrow 2017/2018 \longrightarrow 2038$

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