The Z boson resonance at two loops

Janusz Gluza

in collaboration with

Ievgen Dubovyk, Ayres Freitas, Tord Riemann, Johann Usovitsch

XIV "Loops and Legs"

Sankt Goar, 1 May 2018

50 year of Z-boson physics

COLLEGE OF ARTS AND SCIENCES
CASE WESTERN RESERVE UNIVERSITY

SM@50: The Standard Model At 50 Years

Home | Organizing committee | Registration | Program | Speakers | Accommodation | APS Code | Child Care | Poster | PASCOS

3x2x1 50!

SM@50

The Standard Model at 50 Years: a celebratory symposium will take place in the Physics Department Case Western Reserve University

Speakers

Steven Adler
James "BJ" Bjorken
Alain Blondel
John Butterworth
Norman Christ
Savas Dimopoulos
Henriette Elvang
Pavel Fileviez Perez
Alexei Filippenko
Jerome Friedman
Mary K. Gaillard
David Gross
Gerard 't Hooft
Takaaki Kajita
Rocky Kolb
Bryan W. Lynn
Michael Peskin
Hellen Quinn
Carlo Rubbia
Jurgen Schukraft
George Smoot
Glenn Starkman
Samuel Ting
Bennie F.L. Ward
Steven Weinberg
Mark Wise
Sau Lan Wu

50 years of the Z-boson theory (1967)

S. Weinberg

"A MODEL OF LEPTONS"

VOLUME 19, NUMBER 21 PHYSICAL REVIEW LETTERS 20 NOVEMBER 1967

and

$$\varphi_1 \equiv (\varphi^0 + \varphi^{0\dagger} - 2\lambda)/\sqrt{2} \quad \varphi_2 \equiv (\varphi^0 - \varphi^{0\dagger})/i\sqrt{2}. \quad (5)$$

The condition that φ_1 have zero vacuum expectation value to all orders of perturbation theory tells us that $\lambda^2 \cong M_1^2/2h$, and therefore the field φ_1 has mass M_1 while φ_2 and φ^- have mass zero. But we can easily see that the Goldstone bosons represented by φ_2 and φ^- have no physical coupling. The Lagrangian is gauge invariant, so we can perform a combined isospin and hypercharge gauge transformation which eliminates φ^- and φ_2 everywhere[6] without changing anything else. We will see that G_e is very small, and in any case M_1 might be very large,[7] so the φ_1 couplings will also be disregarded in the following.

The effect of all this is just to replace φ everywhere by its vacuum expectation value

$$\langle\varphi\rangle = \lambda\binom{1}{0}. \quad (6)$$

The first four terms in $\mathfrak{L}$ remain intact, while the rest of the Lagrangian becomes

$$-\tfrac{1}{8}\lambda^2 g^2[(A_\mu{}^1)^2 + (A_\mu{}^2)^2] - \tfrac{1}{8}\lambda^2(gA_\mu{}^3 + g'B_\mu)^2 - \lambda G_e \bar{e}e. \quad (7)$$

We see immediately that the electron mass is λG_e. The charged spin-1 field is

$$W_\mu \equiv 2^{-1/2}(A_\mu{}^1 + iA_\mu{}^2) \quad (8)$$

and has mass

$$M_W = \tfrac{1}{2}\lambda g. \quad (9)$$

The neutral spin-1 fields of definite mass are

$$Z_\mu = (g^2 + g'^2)^{-1/2}(gA_\mu{}^3 + g'B_\mu), \quad (10)$$

$$A_\mu = (g^2 + g'^2)^{-1/2}(-g'A_\mu{}^3 + gB_\mu). \quad (11)$$

Their masses are

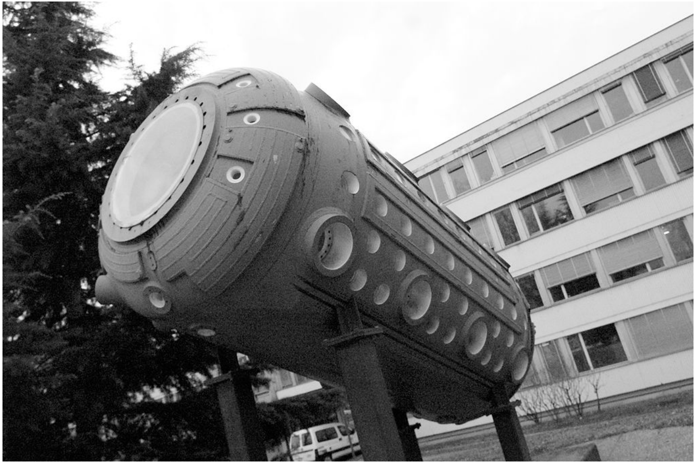
$$M_Z = \tfrac{1}{2}\lambda(g^2 + g'^2)^{1/2}, \quad (12)$$

$$M_A = 0, \quad (13)$$

so A_μ is to be identified as the photon field. The interaction between leptons and spin-1 mesons is

$$\frac{ig}{2\sqrt{2}}\bar{e}\gamma^\mu(1+\gamma_5)\nu W_\mu + \text{H.c.} + \frac{igg'}{(g^2+g'^2)^{1/2}}\bar{e}\gamma^\mu e A_\mu + \frac{i(g^2+g'^2)^{1/2}}{4}\left[\left(\frac{3g'^2-g^2}{g'^2+g^2}\right)\bar{e}\gamma^\mu e - \bar{e}\gamma^\mu\gamma_5 e + \bar{\nu}\gamma^\mu(1+\gamma_5)\nu\right]Z_\mu. \quad (14)$$

And, exactly 45 years of the Z-boson discovery (1973)



Gargamelle

Rich physics

Erler, Freitas, PDG'17

Presently:

Very good agreement

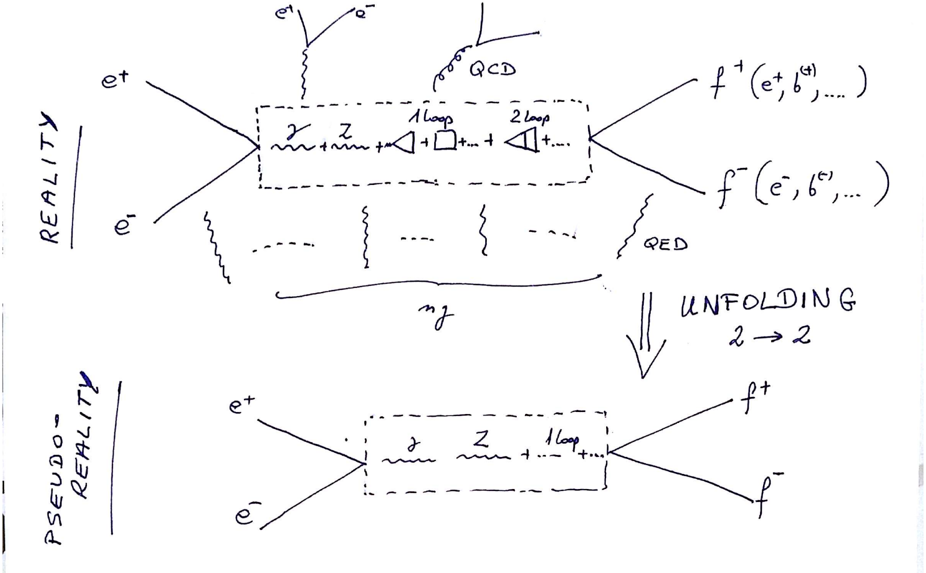
theory — experiment

over large number of EWPOs

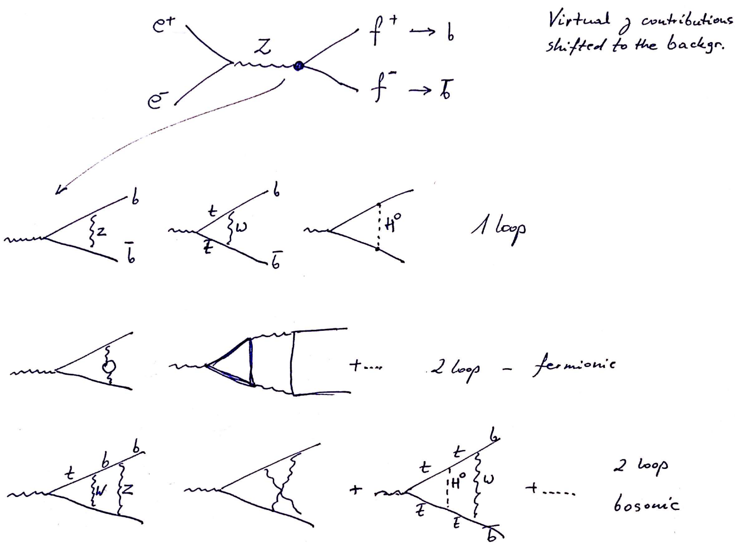
Table 10.5: Principal Z pole observables and their SM predictions (*cf.* Table 10.4). The first $\bar{s}_\ell^2$ is the effective weak mixing angle extracted from the hadronic charge asymmetry, the second is the combined value from the Tevatron [164–166], and the third from the LHC [170–172]. The values of A_e are (i) from A_{LR} for hadronic final states [159]; (ii) from A_{LR} for leptonic final states and from polarized Bhabba scattering [161]; and (iii) from the angular distribution of the τ polarization at LEP 1. The A_τ values are from SLD and the total τ polarization, respectively.

Quantity	Value	Standard Model	Pull
M_Z [GeV]	91.1876 ± 0.0021	91.1880 ± 0.0020	-0.2
Γ_Z [GeV]	2.4952 ± 0.0023	2.4943 ± 0.0008	0.4
$\Gamma(\text{had})$ [GeV]	1.7444 ± 0.0020	1.7420 ± 0.0008	—
$\Gamma(\text{inv})$ [MeV]	499.0 ± 1.5	501.66 ± 0.05	—
$\Gamma(\ell^+\ell^-)$ [MeV]	83.984 ± 0.086	83.995 ± 0.010	—
σ_{had}[nb]	41.541 ± 0.037	41.484 ± 0.008	1.5
R_e	20.804 ± 0.050	20.734 ± 0.010	1.4
R_μ	20.785 ± 0.033	20.734 ± 0.010	1.6
R_τ	20.764 ± 0.045	20.779 ± 0.010	-0.3
R_b	0.21629 ± 0.00066	0.21579 ± 0.00003	0.8
R_c	0.1721 ± 0.0030	0.17221 ± 0.00003	0.0
$A_{FB}^{(0,e)}$	0.0145 ± 0.0025	0.01622 ± 0.00009	-0.7
$A_{FB}^{(0,\mu)}$	0.0169 ± 0.0013		0.5
$A_{FB}^{(0,\tau)}$	0.0188 ± 0.0017		1.5
$A_{FB}^{(0,b)}$	0.0992 ± 0.0016	0.1031 ± 0.0003	-2.4
$A_{FB}^{(0,c)}$	0.0707 ± 0.0035	0.0736 ± 0.0002	-0.8
$A_{FB}^{(0,s)}$	0.0976 ± 0.0114	0.1032 ± 0.0003	-0.5
$\bar{s}_\ell^2$	0.2324 ± 0.0012	0.23152 ± 0.00005	0.7
	0.23185 ± 0.00035		0.9
	0.23105 ± 0.00087		-0.5
A_e	0.15138 ± 0.00216	0.1470 ± 0.0004	2.0
	0.1544 ± 0.0060		1.2
	0.1498 ± 0.0049		0.6
A_μ	0.142 ± 0.015		-0.3
A_τ	0.136 ± 0.015		-0.7
	0.1439 ± 0.0043		-0.7
A_b	0.923 ± 0.020	0.9347	-0.6
A_c	0.670 ± 0.027	0.6678 ± 0.0002	0.1
A_s	0.895 ± 0.091	0.9356	-0.4

Rough scheme for extracting the $Z\bar{f}f$ vertex and EW corrections (1)



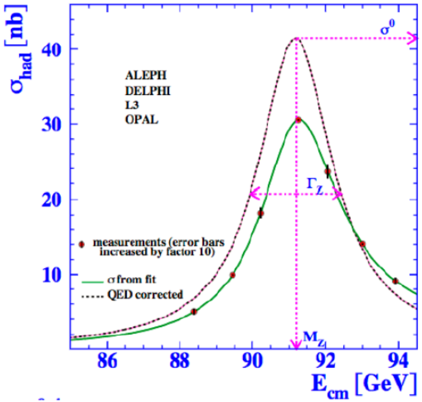
Rough scheme for extracting the $Z\bar{f}f$ vertex and EW corrections (2)



QED unfolding

Altogether $17 \cdot 10^6$ Z-boson decays at LEP

- Cross section : Z mass and width



- ~30% QED corrections (ISR)

EWPOs (electroweak pseudo-observables)

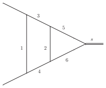
$$\sigma_{peak}^{real} \longrightarrow \begin{cases} \sigma_0 \equiv \sigma_{peak}^{eff.,Born} \\ M_Z, \Gamma_Z, \Gamma_{partial} \\ A_{FB,peak}^{eff.,Born}, A_{LR,peak}^{eff.,Born} \\ R_b, R_\ell \end{cases}$$

- Not got for free! Unfolding of QED — improvements needed for basic LEP programs: KKMC, ZFITTER,...

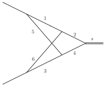
EWPOs & Form Factors

fermionic,bosonic

$$V_\mu^{Zb\bar{b}} = \gamma_\mu[v_b(s) - a_b(s)\gamma_5] = \cdots +$$



$+$



$$+\cdots$$

planar,non−planar

Note approximate factorization of weak couplings

$$A_{F-B} \;=\; \frac{\left[\int_0^1 d\cos\theta - \int_{-1}^0 d\cos\theta\right]\frac{d\sigma}{d\cos\theta}}{\sigma_T} \sim \overbrace{\frac{2a_e v_e}{a_e^2+v_b^2}}^{A_e}\;\overbrace{\frac{2a_b v_b}{a_b^2+v_b^2}}^{A_b} + \text{corrections}$$

$$A_\mathrm{b} = \frac{2\Re e\frac{v_b}{a_b}}{1+\left(\Re e\frac{v_b}{a_b}\right)^2} = \frac{1-4|Q_b|\sin^2\theta_\mathrm{eff}^\mathrm{b}}{1-4|Q_b|\sin^2\theta_\mathrm{eff}^\mathrm{b} + 8Q_b^2(\sin^2\theta_\mathrm{eff}^\mathrm{b})^2}, \quad \sin^2\theta_\mathrm{eff}^\mathrm{b} \longrightarrow F\left(\Re e\frac{v_b}{a_b}\right)$$

Past → present → future

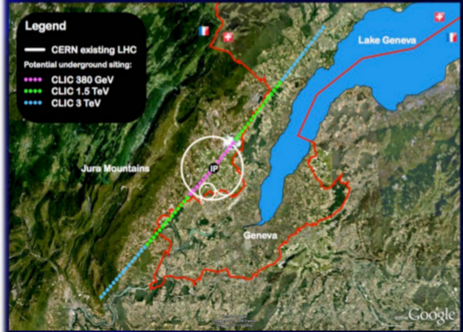
- LEP and SLC studies, the effects of EW quantum corrections became visible in global SM fits:
  m_t, m_H;
- The improved precision - a platform for deep tests of the quantum structure;
- Unprecendented sensitivity to heavy or super-weakly coupled new physics.

Future Linear e^+e^- Colliders

ILC

International Linear Collider,

Kitakami, Japan

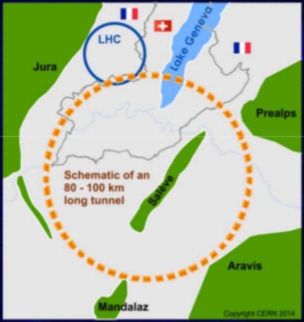


CLIC

Compact Linear Collider,

CERN

Future Circular e^+e^- Colliders

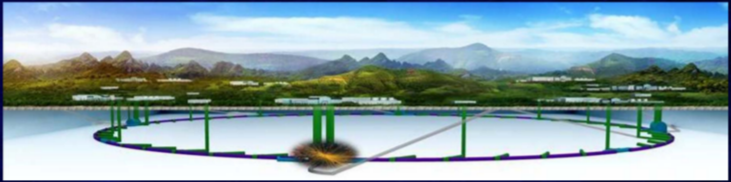


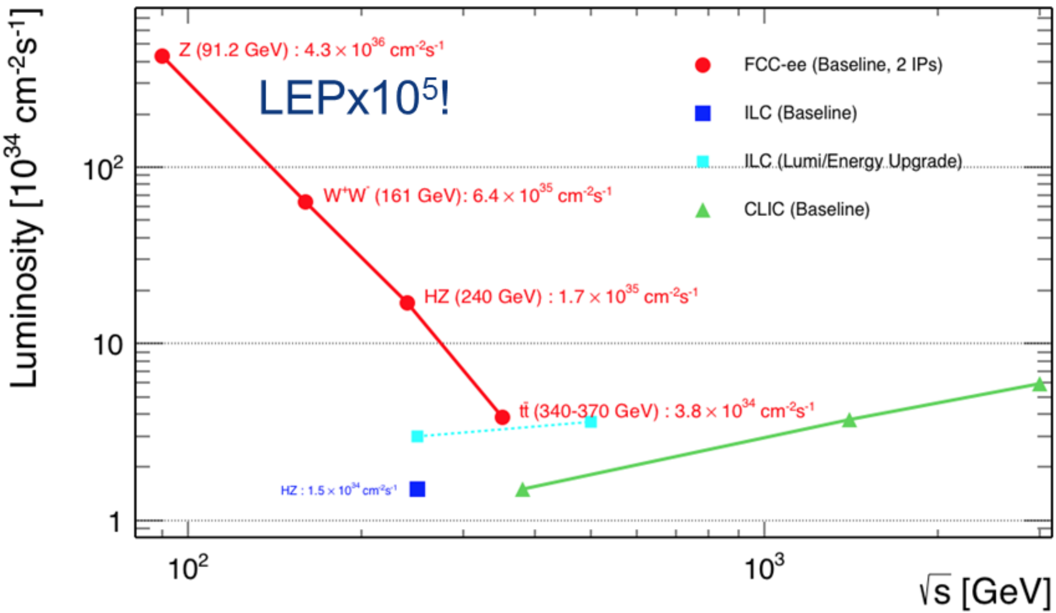
FCC – ee

Future Circular Collider,
CERN

CEPC

Circular Electron Positron Collider,
China




Luminosity [10^{34} cm^{-2}s^{-1}]
Z (91.2 GeV) : 4.3 × 10^{36} cm^{-2}s^{-1}
LEPx10^5!
W$^+$W$^-$ (161 GeV): 6.4 × 10^{35} cm^{-2}s^{-1}
HZ (240 GeV) : 1.7 × 10^{35} cm^{-2}s^{-1}
t$\bar{t}$ (340-370 GeV) : 3.8 × 10^{34} cm^{-2}s^{-1}
HZ : 1.5 × 10^{34} cm^{-2}s^{-1}
FCC-ee (Baseline, 2 IPs)
ILC (Baseline)
ILC (Lumi/Energy Upgrade)
CLIC (Baseline)
10^2
10
1
10^2
10^3
$\sqrt{s}$ [GeV]

LEP uncertainities, A. Freitas: 1604.00406

	Experiment	Theory error	Main source
M_W	80.385 ± 0.015 MeV	4 MeV	α^3, $\alpha^2\alpha_s$
Γ_Z	2495.2 ± 2.3 MeV	0.5 MeV	α^2_{bos}, α^3, $\alpha^2\alpha_s$, $\alpha\alpha_s^2$
σ^0_{had}	41540 ± 37 pb	6 pb	α^2_{bos}, α^3, $\alpha^2\alpha_s$
$R_b \equiv \Gamma_Z^b/\Gamma_Z^{had}$	0.21629 ± 0.00066	0.00015	α^2_{bos}, α^3, $\alpha^2\alpha_s$
$\sin^2\theta^\ell_{eff}$	0.23153 ± 0.00016	4.5×10^{-5}	α^3, $\alpha^2\alpha_s$

This talk: α^2_{bos} results will be shown and discussed

Earlier projections, A. Freitas: 1604.00406

	Measurement error			Intrinsic theory	
	ILC	CEPC	FCC-ee	Current	Future†
M_{W} [MeV]	3–4	3	1	4	1
Γ_{Z} [MeV]	0.8	0.5	**0.1**	**0.5**	**0.2**
R_b $[10^{-5}]$	14	17	6	15	7
$\sin^2 \theta^{\ell}_{\mathrm{eff}}$ $[10^{-5}]$	1	2.3	**0.6**	4.5	**1.5**

Table: Projected experimental and theoretical uncertainties for some electroweak precision pseudo-observables.

† Based on estimations for: $\mathcal{O}(\alpha_{bos}^2)$,$\mathcal{O}(\alpha\alpha_s^2)$, $\mathcal{O}(N_f\alpha^2\alpha_s)$, $\mathcal{O}(N_f^2\alpha^3)$

Published results on EWPOs in the SM @NNLO

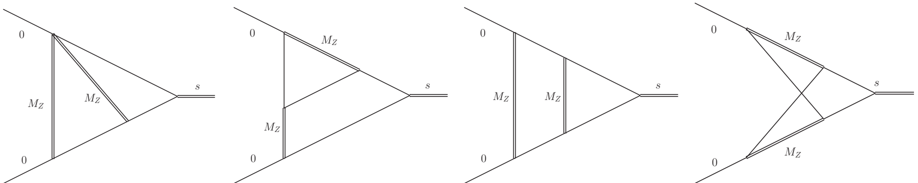
Complete corrections $\Delta r, \sin^2\theta^l_{\rm eff}$:	Freitas, Hollik, Walter, Weiglein: '00
	Awramik,Czakon: '02,Onishchenko,Veretin: '02
	Awramik,Czakon,Freitas,Weiglein: '04
	Awramik,Czakon,Freitas: '06
	Hollik,Meier,Uccirati: '05,'07
	Degrassi,Gambino,Giardino: '14
Fermionic corrections $\sin^2\theta^b_{\rm eff}, a_f, v_f$:	Awramik,Czakon,Freitas,Kniehl: '09
	Czarnecki,Kühn: '96
	Harlander,Seidensticker,Steinhauser: '98
	Freitas: '13,'14
Bosonic corrections $\sin^2\theta^{\rm b}_{\rm eff}$:	Dubovyk, Freitas, JG, Riemann, Usovitsch '16
This talk: Bosonic corrections a_f, v_f :	Dubovyk, Freitas, JG, Riemann, Usovitsch '18

Mellin-Barnes and Sector Decomposition methods are very much complementary

- MB works well for hard threshold, on-shell cases, not many internal masses (more IR); SD more useful for integrals with many internal masses
  $\longrightarrow$ talk by Johann Usovitsch, LL2018
  $\longrightarrow$ JG, Tord Riemann in PoS-LL2016 & DFGRU in PLB'16.

10^{-8} accuracy achieved for **any** self-energy and vertex Feynman integral with one of the methods - in **Minkowskian region**.

Available for several years!



New results for completing NNLO

Input parameters:

Parameter	Value	Parameter	Value
M_Z	91.1876 GeV	$m_b^{\overline{MS}}$	4.20 GeV
Γ_Z	2.4952 GeV	$m_c^{\overline{MS}}$	1.275 GeV
M_W	80.385 GeV	m_τ	1.777 GeV
Γ_W	2.085 GeV	$\Delta\alpha$	0.05900
M_H	125.1 GeV	$\alpha_s(M_Z)$	0.1184
m_t	173.2 GeV	G_μ	1.16638×10^{-5} GeV^{-2}

The 2-loops EWPOs results* for $\mathcal{O}(\alpha^2_{\rm bos})$, **hep-ph/1804.10236**

Γ_i [MeV]	$\Gamma_e, \Gamma_\mu, \Gamma_\tau$	$\Gamma_{\nu_e}, \Gamma_{\nu_\mu}, \Gamma_{\nu_\tau}$	Γ_d, Γ_s	Γ_u, Γ_c	Γ_b	Γ_Z
Born	81.142	160.096	371.141	292.445	369.56	2420.2
$\mathcal{O}(\alpha)$	2.273	6.174	9.717	5.799	3.857	60.22
$\mathcal{O}(\alpha\alpha_{\rm s})$	0.288	0.458	1.276	1.156	2.006	9.11
$\mathcal{O}(N_f^2\alpha^2)$	0.244	0.416	0.698	0.528	0.694	5.13
$\mathcal{O}(N_f\alpha^2)$	0.120	0.185	0.493	0.494	0.144	3.04
$\mathcal{O}(\alpha^2_{\rm bos})$	**0.017**	**0.019**	**0.058**	**0.057**	**0.167**	**0.505**
$\mathcal{O}(\alpha_{\rm t}\alpha_{\rm s}^2,\ \alpha_{\rm t}\alpha_{\rm s}^3,\ \alpha_{\rm t}^2\alpha_{\rm s},\ \alpha_{\rm t}^3)$	0.038	0.059	0.191	0.170	**0.190**	1.20

1. Fun fact of the day: so far all contributions positive.
2. 2016, estimation, bosonic NNLO $\sim 0 \pm 0.1$ MeV

   2018, exact result: 0.505 MeV

* Fixed values of M_W

The 2-loops EWPOs results for $\mathcal{O}(\alpha^2_{\rm bos})$, **hep-ph/1804.10236**

	$\Gamma_{\rm Z}$ [GeV]	$\sigma^0_{\rm had}$ [nb]
Born	2.53601	41.6171
$+\ \mathcal{O}(\alpha)$	2.49770	41.4687
$+\ \mathcal{O}(\alpha\alpha_{\rm s})$	2.49649	41.4758
$+\ \mathcal{O}(\alpha_{\rm t}\alpha_{\rm s}^2,\ \alpha_{\rm t}\alpha_{\rm s}^3,\ \alpha_{\rm t}^2\alpha_{\rm s},\ \alpha_{\rm t}^3)$	2.49560	41.4770
$+\ \mathcal{O}(N_f^2\alpha^2, N_f\alpha^2)$	2.49441	41.4883
$+\ \mathcal{O}(\alpha^2_{\rm bos})$	**[+0.34 MeV]=**2.49475	**[+1.3 pb]=**41.4896

Results for $\Gamma_{\rm Z}$ and $\sigma^0_{\rm had}$, with $M_{\rm W}$ calculated from G_μ using the same order of perturbation theory as indicated in each line.

The 2-loops EWPOs results for $\mathcal{O}(\alpha^2_{\rm bos})$, **hep-ph/1804.10236**

	R_ℓ	R_c	R_b
Born	21.0272	0.17306	0.21733
$+\ \mathcal{O}(\alpha)$	20.8031	0.17230	0.21558
$+\ \mathcal{O}(\alpha\alpha_{\rm s})$	20.7963	0.17222	0.21593
$+\ \mathcal{O}(\alpha_{\rm t}\alpha^2_{\rm s},\ \alpha_{\rm t}\alpha^3_{\rm s},\ \alpha^2_{\rm t}\alpha_{\rm s},\ \alpha^3_{\rm t})$	20.7943	0.17222	0.21593
$+\ \mathcal{O}(N^2_f\alpha^2, N_f\alpha^2)$	20.7512	0.17223	0.21580
$+\ \mathcal{O}(\alpha^2_{\rm bos})$	20.7516	0.17222	0.21585

Results for the ratios R_ℓ, R_c and R_b, with $M_{\rm W}$ calculat ed from G_μ to the same order as indicated in each line.

Updates for error estimations

- Theory error estimate is not well defined, ideally $\Delta_{\mathrm{th}} \ll \Delta_{\mathrm{exp}}$
- Common methods:
  - Count prefactors (α, N_c, N_f, ...)
  - Extrapolation of perturbative series
  - Renormalization scale dependence
  - Renormalization scheme dependence
- Also parametric error from external inputs (m_{t}, m_b, α_{s}, $\Delta\alpha_{\mathrm{had}}$, ...)

see, Ayres Freitas: 1604.00406

E.g.: Intrinsic theory error estimation for Γ_Z, 1804.10236 [1604.00406]

1. Geometric series

$$\delta_1 : \mathcal{O}(\alpha^3) - \mathcal{O}(\alpha_t^3) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha^2) \sim 0.20 \text{ MeV } [0.26 \text{ MeV}]$$

$$\delta_2 : \mathcal{O}(\alpha^2\alpha_s) - \mathcal{O}(\alpha_t^2\alpha_s) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha\alpha_s) \sim 0.21 \text{ MeV } [0.3 \text{ MeV}]$$

$$\delta_3 : \mathcal{O}(\alpha\alpha_s^2) - \mathcal{O}(\alpha_t\alpha_s^2) \sim \frac{\mathcal{O}(\alpha\alpha_s) - \mathcal{O}(\alpha_t\alpha_s)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha\alpha_s) \sim 0.23 \text{ MeV}$$

$$\delta_4 : \mathcal{O}(\alpha\alpha_s^3) - \mathcal{O}(\alpha_t\alpha_s^3) \sim \frac{\mathcal{O}(\alpha\alpha_s) - \mathcal{O}(\alpha_t\alpha_s)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha\alpha_s^2) \sim 0.035 \text{ MeV}$$

$$\delta_5 : \mathcal{O}(\alpha_{bos}^2) \sim \mathcal{O}(\alpha_{bos})^2 \sim \mathbf{0.1} \text{ MeV } \underline{[\text{Now we know it!}]}$$

Total: $\delta\Gamma_Z = \sqrt{\sum_{i=1}^{5} \delta_i^2} \sim \mathbf{0.4}$ **MeV** [0.5 MeV]

Summary: estimations for higher order EW and QCD corrections

δ_1 :	δ_2 :	δ_3 :	δ_4 :	δ_5 :	$\delta\Gamma_Z$ [MeV]
$\mathcal{O}(\alpha^3)$	$\mathcal{O}(\alpha^2\alpha_s)$	$\mathcal{O}(\alpha\alpha_s^2)$	$\mathcal{O}(\alpha\alpha_s^3)$	$\mathcal{O}(\alpha_{bos}^2)$	$= \sqrt{\sum_{i=1}^5 \delta_i^2}$
TH1 (estimated error limits from geometric series of perturbation)					
0.26	0.3	0.23	0.035	0.1	0.5
TH1-new (estimated error limits from geometric series of perturbation)					
0.2	0.21	0.23	0.035	$< 10^{-4}$	**0.4**

δ'_1 :	δ'_2 :	δ'_3 :	δ_4 :		$\delta\Gamma_Z$ [MeV]
$\mathcal{O}(N_f^{\leq 1}\alpha^3)$	$\mathcal{O}(\alpha^3\alpha_s)$	$\mathcal{O}(\alpha^2\alpha_s^2)$	$\mathcal{O}(\alpha\alpha_s^3)$		$\sqrt{\delta_1'^2 + \delta_2'^2 + \delta_2'^3 + \delta_4^2}$
TH2 (extrapolation through prefactor scaling)					
0.04	0.1	0.1	0.035	10^{-4}	**0.15**

Crucial issue: accuracy of calculations

For 2-loops we maintained 4 digits for EWPOs.

A calculation of the radiative corrections $\delta_1 \div \delta_4$ and $\delta'_1 \div \delta'_3$ with a 10% accuracy (corresponding to two significant digits) should suffice to meet future experimental demands.

Minimal precision of 3-loop EW calculations:

1. Calculating N^3LO with 10% accuracy (two digits), we can replace intrinsic error estimation $\delta\Gamma_Z = \sqrt{\sum_{i=1}^{5} \delta_i^2} \sim 0.4$ MeV by $\delta\Gamma_Z = \sqrt{\sum_{i=1}^{5} (\delta_i/10)^2} \sim 0.04$ MeV.

1. The requirement of FCC-ee$^{exper.\ error}(\Gamma_Z) \sim 0.1$ MeV can be met and the condition

$$\delta[\mathrm{FCCee}^{\mathrm{theor.}}(\Gamma_Z)] \;\sim 0.04\ \mathrm{MeV} < \delta[\mathrm{FCCee}^{exper.}(\Gamma_Z)] \;\sim 0.1\ \mathrm{MeV}$$

will be fullfilled.

Estimations for total values of missing EWPOs

	$\delta\Gamma_Z$ [MeV]	δR_l $[10^{-4}]$	δR_b $[10^{-5}]$	$\sin^2\theta^{\rm l}_{\rm eff}$ $[10^{-5}]$	$\sin^2\theta^{\rm b}_{\rm eff}$ $[10^{-5}]$	$\sigma^0_{\rm had}$ [pb]
EXP-FCCee	0.1	$2 \div 20$	$2 \div 6$	6	70	4
TH1*	0.4	60	10	4.5	5	6
TH2*	0.15	60	5	1.5	$1.5 \div 2$	6

TH1 - estimates from geometric series (3-loops)
TH2 - estimates from prefactor scaling (beyond 3-loops)

* **10% knowledge (2 digits) of the error would decrease numbers by factor 10**

And this should be the goal for future $\geq N^3LO$ calculations

Conclusions on Z-lineshape and EWPOs for next years - theory

- NNLO EWPOs completed;
- Strong demand from FCC-ee to the theory on precision;
- Future $\geq$ NNNLO calculations must be done with at least 10% accuracy, e.g. $\mathcal{O}(\alpha\alpha_s^2), \mathcal{O}(N_f\alpha^2\alpha_s), \mathcal{O}(N_f^2\alpha^3)$, $\mathcal{O}(N_f^{\leq 1}\alpha^3)$, $\mathcal{O}(\alpha^3\alpha_\mathrm{s})$, $\mathcal{O}(\alpha^2\alpha_\mathrm{s}^2)$;
- **We have tools for that;**
- To be on the safe side, we would like to have **at least 2 independent calculations**;
- Still, a lot work is ahead, for success and efficiency, **we need steady progress in numerical and also (semi)analytical approaches** in multiloop calculations

Backup slides

Discussion of NNNLO accuracy.

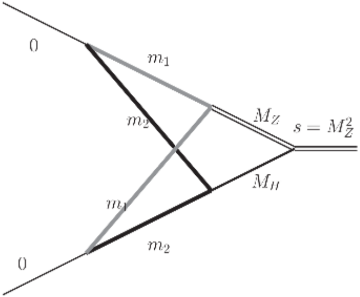
Two factors play role:

- Number of diagrams
- Their complexity

Goal: at least 2-digits accuracy for EWPOs.

We estimate it to be possible, even from present perspective.

2-loops $\longrightarrow$ 3-loops



$$m_1 = M_t, m_2 = M_W$$

The integrals contain up to three dimensionless parameters

$$\left\{ \frac{M_H^2}{M_Z^2}, \frac{M_W^2}{M_Z^2}, \frac{m_t^2}{M_Z^2}, \frac{(M_Z + i\varepsilon)^2}{M_Z^2} \right\}$$

2-loops $\longrightarrow$ 3-loops

- The standard model prediction for the effective weak mixing angle can be written as

$$\sin^2\theta^{\mathrm{b}}_{\mathrm{eff}} = \left(1 - \frac{M_W^2}{M_Z^2}\right)(1+\Delta\kappa_{\mathrm{b}})$$

- The bosonic electroweak two-loop corrections amount to

$$\boxed{\Delta\kappa_{\mathrm{b}}^{(\alpha^2,\mathrm{b}os)} = -0.9855\times 10^{-4}}$$

DFGRU, Phys.Lett. B762 (2016) 184

Collection of radiative corrections: Full stabilization at 10^{-4}!

Order	Value $[10^{-4}]$	Order	Value $[10^{-4}]$
α	468.945	$\alpha_t^2\alpha_s$	1.362
$\alpha\alpha_s$	-42.655	α_t^3	0.123
α^2_{ferm}	3.866	$\alpha_t\alpha_s^2$	-7.074
α^2_{bos}	**-0.9855**	$\alpha_t\alpha_s^3$	-1.196

$\pm 0.001 \overset{!}{\longrightarrow}$

Table: *Comparison of different orders of radiative corrections to* $\Delta\kappa_{\mathrm{b}}$.
Input Parameters: M_Z, Γ_Z, M_W, Γ_W, M_H, m_t, α_s *and* $\Delta\alpha$

- one-loop contributions [Akhundov, Bardin, Riemann, 1986] [Beenakker, Hollik, 1988]
- two-loop fermionic contributions [Awramik, Czakon, Freitas, Kniehl, 2009]
- two-loop bosonic contributions [Dubovyk, Freitas, JG, Riemann, Usovitsch, 2016]

Partial higher-order corrections

$\mathcal{O}(\alpha_{\mathrm{t}}\alpha_{\mathrm{s}}^2)$ Avdeev: 1994, Chetyrkin: 1995
$\mathcal{O}(\alpha_{\mathrm{t}}\alpha_{\mathrm{s}}^3)$ Schroder: 2005, Chetyrkin: 2006, Boughezal: 2006
$\mathcal{O}(\alpha^2\alpha_{\mathrm{t}})$ and $\mathcal{O}(\alpha_{\mathrm{t}}^3)$ vanderBij: 2000, Faisst: 2003

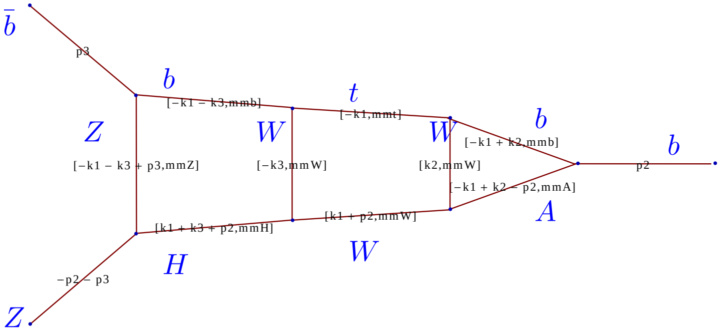
3-loops. Basic bookkeeping

$Z \to b\bar{b}$			
Number of topologies	1 loop	2 loops	3 loops
	1	14 $\to^{(\mathbf{A})}$ 7 $\to^{(\mathbf{B})}$ **5**	211 $\to^{(\mathbf{A})}$ 84 $\to^{(\mathbf{B})}$ **50**
Number of diagrams	15	2383 $\to^{(\mathbf{A,B})}$ **1114**	490387 $\to^{(\mathbf{A,B})}$ **120187**
Fermionic loops	0	150	**17580**
Bosonic loops	15	**964**	102607
Planar diagrams	1T/15D	4T/981D	35T/84059D
Non-planar diagrams	0	1T/133D	15T/36128D

Table: Some statistical overview for $Z \to b\bar{b}$ multiloop studies. At 3 loops there are in total almost half a million of diagrams present. After basic refinements (A) and (B) about 10^5 genuine 3-loop vertex diagrams remain. In (A) tadpoles and products of lower loops are excluded, in (B) symmetries of topologies are taken into account.

A complete zoo of heavy particles m_t, m_W, m_Z, m_H @NNNLO level

MB: ϵ^0**[8-dim]**, $1/\epsilon$**[7-dim]**; SD: ϵ^0**[8-dim]** , $1/\epsilon$**[7-dim]**;



At 2-loops up to three dimensionless parameters (all 4 at 3-loops):

$$\left\{ \frac{M_H^2}{M_Z^2}, \frac{M_W^2}{M_Z^2}, \frac{m_t^2}{M_Z^2}, \frac{(M_Z + i\varepsilon)^2}{M_Z^2} \right\}$$

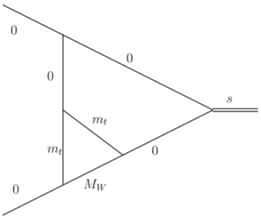
Sector decomposition

FIESTA 3 [A.V.Smirnov, 2014], SecDec 3 [Borowka, et. al., 2015] and pySecDec [Borowka, et. al., 2017]

Mellin-Barnes integral approach

- With AMBRE 2 [Gluza, et. al., 2011] (AMBRE 3 [Dubovyk, et. al., 2015]) we derive Mellin-Barnes representations for planar (non-planar) topologies. One may use PlanarityTest [Bielas, et. al, 2013] for automatic identification.
- Expansion in terms of $\epsilon = (4 - D)/2$ is done with MB [Czakon, 2006], MBresolve [A. Smirnov, V. Smirnov, 2009], barnesroutines (D. Kosower).
- For the numerical treatment of massive Mellin-Barnes integrals with Minkowskian regions, the package MBnumerics is being developed since 2015.

soft7 ϵ^0:[MB - 3 dim] [SD - 5 dim], ϵ^{-1}:[MB - 2 dim] [SD - 4 dim], ϵ^{-2}:[MB - 1 dim] [SD - 3 dim]



MB	$0.06026648655769\boldsymbol{9}\,\epsilon^{-2}$	
SD - 90 Mio	$0.060266486\boldsymbol{5}\,\epsilon^{-2}$	
MB	$(-0.031512489\boldsymbol{0}3$	$+0.189332751\boldsymbol{4}2i)\,\epsilon^{-1}$
SD - 90 Mio	$(-0.03151248\boldsymbol{1}6$	$+0.18933271\boldsymbol{6}96i)\,\epsilon^{-1}$
MB 1	$(-0.228231867\boldsymbol{5}\boldsymbol{1}1$	$-0.0882479456\boldsymbol{9}1i) + \mathcal{O}(\epsilon)$
MB 2	$(-0.22823186755\boldsymbol{5}1$	$-0.088247945\boldsymbol{7}39i) + \mathcal{O}(\epsilon)$
SD - 90 Mio	$(-0.228226\boldsymbol{5}3$	$-0.0882459\boldsymbol{6}6i) + \mathcal{O}(\epsilon)$
SD - 15 Mio	$(-0.2281\boldsymbol{6}2$	$-0.0882\boldsymbol{0}9i) + \mathcal{O}(\epsilon)$

Intermezzo: $1997 \longrightarrow 2017/2018 \longrightarrow 2038$

References for EWPOs corrections I

$\mathcal{O}(\alpha)$ EW complete corrections:

[1] A. A. Akhundov, D. Y. Bardin and T. Riemann, Electroweak One Loop Corrections to the Decay of the Neutral Vector Boson, Nucl. Phys. B **276** (1986) 1. doi.org/10.1016/0550-3213(86)90014-3.

$\mathcal{O}(\alpha\alpha_s)$ QCD corrections:

[1] A. Djouadi, C. Verzegnassi, Virtual very heavy top effects in LEP/SLC precision measurements, Phys. Lett. B195 (1987) 265–271. doi:10.1016/0370-2693(87)91206-8.

[2] A. Djouadi, $O(\alpha\alpha_s)$ vacuum polarization functions of the standard model gauge bosons, Nuovo Cim. A100 (1988) 357. doi:10.1007/BF02812964.

[3] B. A. Kniehl, Two loop corrections to the vacuum polarizations in perturbative QCD, Nucl. Phys. B347 (1990) 86–104. doi:10.1016/0550-3213(90)90552-O.

[4] B. A. Kniehl, A. Sirlin, Dispersion relations for vacuum polarization functions in electroweak physics, Nucl. Phys. B371 (1992) 141–148. doi:10.1016/0550-3213(92)90232-Z.

[5] A. Djouadi, P. Gambino, Electroweak gauge bosons selfenergies: Complete QCD corrections, Phys. Rev. D49 (1994) 3499–3511, Erratum: Phys. Rev. D53 (1996) 4111. arXiv:hep-ph/9309298, doi:10.1103/PhysRevD.49.3499,10.1103/PhysRevD.53.4111.

[6] J. Fleischer, O. Tarasov, F. Jegerlehner, P. Raczka, Two loop $\mathcal{O}(\alpha_s G_\mu m_t^2)$ corrections to the partial decay width of the Z^0 into $b\bar{b}$ final states in the large top mass limit, Phys. Lett. B293 (1992) 437–444. doi:10.1016/0370-2693(92)90909-N.

[7] G. Buchalla, A. Buras, QCD corrections to the $\bar{s}dZ$ vertex for arbitrary top quark mass, Nucl. Phys. B398 (1993) 285–300. doi:10.1016/0550-3213(93)90110-B.

References for EWPOs corrections II

[8] G. Degrassi, Current algebra approach to heavy top effects in $Z \to b + \bar{b}$, Nucl. Phys. B407 (1993) 271–289. arXiv:hep-ph/9302288, doi:10.1016/0550-3213(93)90058-W.

[9] K. Chetyrkin, A. Kwiatkowski, M. Steinhauser, Leading top mass corrections of order $O(\alpha\alpha_s m_t^2/M_W^2)$ to partial decay rate $\Gamma(Z \to b\bar{b})$, Mod. Phys. Lett. A8 (1993) 2785–2792. doi:10.1142/S0217732393003172.

[10] A. Czarnecki, J. H. Kühn, Nonfactorizable QCD and electroweak corrections to the hadronic Z boson decay rate, Phys. Rev. Lett. 77 (1996) 3955–3958. arXiv:hep-ph/9608366, doi:10.1103/PhysRevLett.77.3955.

[11] R. Harlander, T. Seidensticker, M. Steinhauser, Complete corrections of order $O(\alpha\alpha_s)$ to the decay of the Z boson into bottom quarks, Phys. Lett. B426 (1998) 125–132. arXiv:hep-ph/9712228, doi:10.1016/S0370-2693(98)00220-2.

Partial higher order corrections of order $\mathcal{O}(\alpha_t\alpha_s^2)$:

[1] L. Avdeev, J. Fleischer, S. Mikhailov, O. Tarasov, $O(\alpha\alpha_s^2)$ correction to the electroweak ρ parameter, Phys. Lett. B336 (1994) 560–566, Erratum: Phys. Lett. B349 (1995) 597. arXiv:hep-ph/9406363, doi:10.1016/0370-2693(94)90573-8.

[2] K. Chetyrkin, J. H. Kühn, M. Steinhauser, Corrections of order $O(G_F M_t^2 \alpha_s^2)$ to the ρ parameter, Phys. Lett. B351 (1995) 331–338. arXiv:hep-ph/9502291, doi:10.1016/0370-2693(95)00380-4.

Partial higher order corrections of order $\mathcal{O}(\alpha_t\alpha_s^3)$:

[1] Y. Schröder, M. Steinhauser, Four-loop singlet contribution to the ρ parameter, Phys. Lett. B622 (2005) 124–130. arXiv:hep-ph/0504055, doi:10.1016/j.physletb.2005.06.085.

[2] K. G. Chetyrkin, M. Faisst, J. H. Kühn, P. Maierhofer, C. Sturm, Four-loop QCD corrections to the ρ parameter, Phys. Rev. Lett. 97 (2006) 102003. arXiv:hep-ph/0605201, doi:10.1103/PhysRevLett.97.102003.

References for EWPOs corrections III

[3] R. Boughezal, M. Czakon, Single scale tadpoles and $O(G_F m_t^2 \alpha_s^3)$ corrections to the ρ parameter, Nucl. Phys. B755 (2006) 221–238. arXiv:hep-ph/0606232.

Partial higher order corrections of orders $\mathcal{O}(\alpha_t^2 \alpha_s)$ and $\mathcal{O}(\alpha_t^3)$:

[1] J. J. van der Bij, K. G. Chetyrkin, M. Faisst, G. Jikia, T. Seidensticker, Three loop leading top mass contributions to the ρ parameter, Phys. Lett. B498 (2001) 156–162. arXiv:hep-ph/0011373, doi:10.1016/S0370-2693(01)00002-8.

[2] M. Faisst, J. H. Kühn, T. Seidensticker, O. Veretin, Three loop top quark contributions to the ρ parameter, Nucl. Phys. B665 (2003) 649–662. arXiv:hep-ph/0302275, doi:10.1016/S0550-3213(03)00450-4.

EW SM theory at loops, an example ($\Delta_{ef} \neq 0$)

$$\left\{ \begin{array}{l} \Gamma_Z, \Gamma_{partial} \\ \\ A_{FB,peak}^{eff.,Born}, A_{LR,peak}^{eff.,Born} \\ \\ R_b, R_\ell, \ldots \end{array} \right. \longrightarrow \left\{ \begin{array}{l} v_{\ell,\nu,u,d,b}^{eff} \\ \\ a_{\ell,\nu,u,d,b}^{eff} \\ \\ \sin^2\theta_{\rm eff}^{\rm b}, \sin^2\theta_{\rm eff}^{lept} \end{array} \right.$$

e.g. : improvements needed for subtle corrections $\Delta_{1,2}$ (e.g. boxes, **2L-boxes**)

$$A_{FB,peak}^{eff.,Born} = \frac{2\Re e\left[\frac{v_e a_e^*}{|a_e|^2}\right] 2\Re e\left[\frac{v_f a_f^*}{|a_f|^2}\right]}{\left(1+\frac{|v_e|^2}{|a_e|^2}\right)\left(1+\frac{|v_f|^2}{|a_f|^2}\right)} + \Delta_1 - \Delta_2 \simeq \frac{3}{4} A_e A_f,$$

$$\Delta_1 = 2\Re e\left[\Delta_{ef}\right], \; \Delta_2 = |\Delta_{ef}|^2 + 2\Re e\left[\frac{v_e a_e^*}{|a_e|^2}\frac{v_f a_f^*}{|a_f|^2}\Delta_{ef}^*\right],$$

$$\Delta_{ef} = 16|Q_e Q_f| s_W^4 (\kappa_{ef} - \kappa_e \kappa_f)$$