# Atom based tests of the Bell inequalities - the Legacy of John Bell continues....

Edward S. Fry* and Thomas Walther
*Department of Physics, Texas A&M University, College Station, TX 77843-4242, U.S.A.*

(Dated: April 12, 2001)

After a brief historical review including a discussion of loopholes associated with previous tests of the Bell inequalities, we review the loophole-free atom based experiment we have underway. It tests the strong Bell-Clauser-Horne inequality and is based on the photo-dissociation of $^{199}\mathrm{Hg}_2$ dimer that is in a nuclear spin-1/2 entangled state. This experiment will be compared to other recent atom based tests.

This paper is dedicated to John Bell whose work not only inspired the experiments discussed in this paper but has also had a far-reaching influence on our understanding of the interpretation of quantum mechanics.

## I. HISTORICAL OVERVIEW

In a period of 30 years at the beginning of the last century the world of Physics was revolutionized. Now, one hundred years after Planck's quantum hypothesis, we continue to be puzzled by the wonders of quantum mechanics. We have learned to apply concepts such as entanglement to problems in quantum information [1, 2]. Other examples of emerging applications that have arisen in the last couple of years include quantum cryptography [3] and teleportation [4–6]. Nevertheless, the article published in 1935 by Albert Einstein, Boris Podolsky, and Nathan Rosen (generally referred to as EPR) continues to fascinate us [7]. EPR argued by means of a *gedanken*experiment that quantum mechanics is not a "complete" theory. This incomplete description could presumably be avoided by postulating the presence of some hidden variables (HV) that would permit deterministic predictions for microscopic events. The relation of quantum mechanics to HV could be considered analogous to the relation of thermodynamics to statistical mechanics. For example, in a gas the thermodynamic quantities, temperature and pressure, can be understood in terms of microscopic (hidden) values of the speeds and directions of the individual atoms. In addition, the HV would eliminate concerns about non-locality.

For many years the discussion was purely philosophical in nature. However, in 1964, John Bell made a giant step forward [8]. Bell revisited the EPR experiment and considered a version based on the entanglement of spin-1/2 particles that was introduced by Bohm [9]. The latter has conceptual advantages in comparison to the original EPR experiment that involves entanglement in position and momentum. The wave function of such a system can be written as

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\Big( |\uparrow\rangle_1 |\downarrow\rangle_2 - |\downarrow\rangle_1 |\uparrow\rangle_2 \Big). \tag{1}$$

Thus, for a given two particle system, if particle one is found to have spin up (↑) in some spatial direction, particle two must necessarily have spin down (↓) in that direction and vice versa. Specifically, quantum mechanics predicts that the total spin of this two particle system is zero, and that for measurement of both components in one specific direction, the results will be opposite with unit probability. However, quantum mechanically it is not possible to predict the absolute orientation of either spin, or to even predict absolute values for two of the components of the spin of a particle.

John Bell showed: (1) Hidden-variable theories can exist in the context of Bohm's version of the EPR experiment. (2) **Any** hidden-variable theory satisfying a physically reasonable condition of locality (LHV theory, e.g. a classical theory) will yield statistical predictions which must satisfy restrictions for certain correlated phenomena such as those involving entangled states (cf. Eq. (1)). In other words, classical theories put upper bounds on measurements of the statistical correlations between the spin components of the two particles. These restrictions can be cast into the form of inequalities, which are now generally referred to as Bell inequalities. Thus, at least in principle, Bell's work made it possible for the first time to experimentally distinguish between the LHV and quantum mechanical pictures. It should be emphasized that no specifics of an LHV theory are involved, only its existence. Thus a test of a Bell inequality is general and leads to discrimination between any LHV and quantum mechanics.

Bell's original formulation of the inequalities was idealized and not readily suited to realistic experimental conditions. Several researchers have since formulated other versions of the Bell inequalities [10] that are more experimentally amenable. Well known examples are the Clauser-Horne-Shimony-Holt (CHSH) inequality [11] and the Bell-Clauser-Horne (BCH) inequality [12]. The CHSH inequality is

*Electronic address: fry@physics.tamu.edu

$$S(\theta_1,\theta_2,\theta_1',\theta_2') = |E(\theta_1,\theta_2) - E(\theta_1',\theta_2)| + E(\theta_1,\theta_2') + E(\theta_1',\theta_2') \leq 2 \quad (2)$$

where $\theta_1$, $\theta_1'$ are two values of angles for the first spin analyzer and $\theta_2$, $\theta_2'$ are two values of angles for the second spin analyzer. In Eq. (2) the quantities $E(\theta_1,\theta_2)$ are expectation values,

$$E(\theta_1,\theta_2) = \frac{N_{\uparrow\uparrow}(\theta_1,\theta_2) - N_{\uparrow\downarrow}(\theta_1,\theta_2) - N_{\downarrow\uparrow}(\theta_1,\theta_2) + N_{\downarrow\downarrow}(\theta_1,\theta_2)}{N_{\rm total}}, \quad (3)$$

where $N_{\uparrow\downarrow}(\theta_1,\theta_2)$ is the number of pairs of particles for which analyzer 1 (oriented in direction $\theta_1$) would give spin up for particle 1 and analyzer 2 (oriented in direction $\theta_2$) would give spin down for particle 2. Definitions for the other $N$'s are analogous. $N_{\rm total}$ is the total number of pairs of particles. The $N$'s denote the number of pairs of particles rather than actual detection events. Or in other words, the $N$'s mean emergence from the analysers, not detection. In this regard Eq. 2 is not an experimentally testable inequality. However, if the detectors have 100% efficiency, detection and emergence from the analyzer are identical and the inequality is testable, e.g. the Boulder experiment [13]. By making the CHSH assumption (the detection efficiency is independent of analyzer orientation), the $E(\theta_1,\theta_2)$ can be formulated in terms of experimental coincidence detection rates.

One can also make the alternative CH assumption that the probability of detection with the analyzer in place is less than or equal to the corresponding probability with the analyzer removed. From a practical standpoint, employing either the CHSH or the CH assumption enables one to obtain inequalities that depend only on coincidence rates (”simultaneous” detection of both members of an entangled pair of particles) and that do not depend on the singles rates for detection of particles at only one of the detectors. As a consequence, these inequalities are independent of detection efficiencies. Freedman has obtained such an inequality in a particularly simple form [14],

$$\frac{|R(22.5^\circ) - R(67.5^\circ)|}{R_0} \leq \frac{1}{4} \quad (4)$$

where $R(22.5^\circ)$ and $R(67.5^\circ)$ are coincidence rates with angles 22.5° and 67.5°, respectively, between the polarization transmission directions of the polarizers for each photon, and $R_0$ is the coincidence rate with both polarizers removed. By contrast, the BCH inequality is

$$S(\theta_1,\theta_2,\theta_1',\theta_2') = \frac{R_{\uparrow\uparrow}(\theta_1,\theta_2) - R_{\uparrow\uparrow}(\theta_1,\theta_2') + R_{\uparrow\uparrow}(\theta_1',\theta_2) + R_{\uparrow\uparrow}(\theta_1',\theta_2')}{R_{1\uparrow}(\theta_1') + R_{2\uparrow}(\theta_2)} \leq 1, \quad (5)$$

where a typical $R_{\uparrow\uparrow}(\theta_1,\theta_2)$ denotes the experimental coincidence count rates when both particles are detected with spin up in the directions $\theta_1$ and $\theta_2$, respectively; $R_{1\uparrow}(\theta_1')$ and $R_{2\uparrow}(\theta_2)$ are singles count rates at detectors 1 and 2, respectively. The BCH inequality is especially important because it provides a direct constraint on the experimentally accessible (observed) detection count rates; it does not require any additional assumptions for experimental implementation. However, it depends on the ratio of coincidence rates to singles rates and is therefore proportional to the detector efficiency. (By comparison, Eq. (4) depends on the ratio of coincidence rates to coincidence rates and thus the detector efficiency in the numerator and denominator of this ratio cancel). Consequently, for low detection efficiency, the inequality (Eq. 5) is always satisfied by the quantum mechanical predictions; this is also recognized by noting that the singles rates in the denominator are large compared to the coincidence rates in the numerator. Therefore, Eq. (5) can only be used for a definitive test if the detectors have very high efficiencies. Note also, that it is not necessary to simultaneously measure both projections in the case of the BCH inequality.

Both Eq. (2) and Eq. (5) apply to deterministic as well as stochastic (a surprising result) theories, that are based on local hidden variables [10]. For a definitive loophole free experiment, the BCH inequality [12] should be tested. Since the BCH inequality contains no additional assumptions, it provides the strongest possible test. It is remarkable that this inequality has actually not yet been tested (cf. section III).

The principle problem for experimental tests of the BCH inequality lies in the fact that experimental imperfections generally preclude a definitive test, i.e. they tend to shift the result such that the Bell inequalities are no longer violated by the quantum mechanical predictions.

## II. THE BELL-CLAUSER-HORNE INEQUALITY

Since our experiment will test the BCH inequality, we will briefly discuss the quantum mechanical prediction for this inequality as it applies to the experimentally relevant situation of our experiment (cf. Section IV C).

The test of the strong BCH-inequality requires the measurement of coincidence rates $R_{\uparrow\uparrow}(\theta_1, \theta_2)$ for the simultaneous detection of one particle of the entangled pair at detector 1 with spin-up ("↑") in the direction $\theta_1$ and of the other atom at detector 2 with spin-up ("↑") in the direction $\theta_2$; the singles rates $R_{i\uparrow}(\theta_i)$, which are defined as the rate of detection of particles with spin-up in the direction $\theta_i$ at detector $i$, where $i$=1 or 2 must also be measured. The parameter for which the pair is entangled (i.e. polarization, time-position, or spin) or the type of particle (i.e. photon or atom) that is entangled is not significant to the argument. However, we will concentrate on atomic systems with an entanglement of the spin according to Eq. (1).

The BCH inequality is formulated in terms of the ratio of coincidence rates to singles rates for four different combinations of angles for the spin measurement [10, 12]. The quantum mechanical predictions including experimental imperfections for the BCH inequality is given by [15, 16]:

$$R_{\uparrow\uparrow}(\theta_1, \theta_2) = \eta^2 f g N \frac{1}{4}\Big\{\varepsilon_+^2 - \varepsilon_-^2 F \cos(\theta_1 - \theta_2)\Big\} \quad (6)$$

$$R_{1\uparrow}(\theta_1) = R_{2\uparrow}(\theta_2) = \frac{\eta f N}{2}\varepsilon_+ \quad (7)$$

where $\eta$ is the detector efficiency for the atoms and is assumed to be the same for both detectors; $\varepsilon_+$ and $\varepsilon_-$ are defined as $\varepsilon_+ = \varepsilon_M + \varepsilon_m$ and $\varepsilon_- = \varepsilon_M - \varepsilon_m$ , respectively, where $\varepsilon_M$ is the transmission of the analyzers for one spin component and $\varepsilon_m$ the leakage through the analyzer for the other spin component; $\varepsilon_m$ and $\varepsilon_M$ are positive definite and are assumed to be identical for both analyzers. Specifically, $\varepsilon_+$ and $\varepsilon_-$ are measures of the capability to discriminate between the two spin components in a measurement. For optimum discrimination, $\varepsilon_+$ and $\varepsilon_-$ are equal to unity. The two detectors are assumed symmetric; they have identical geometries and $f$ is the detector acceptance solid angle; $g$ is the conditional probability that if one of the atoms from a dissociated dimer enters the aperture of one detector, then the other atom from that dimer enters the aperture of the other detector; $N$ is the total number of dissociating dimers per unit time; $F$ is a measure for the purity of the entangled state.

In an actual experiment the goal will be to choose the angles $\theta_1$, $\theta_2$, $\theta_1'$, and $\theta_2'$ such that when the quantum mechanical predictions Eqs. 6 and 7 are used in Eq. (5), the resulting $S_{\rm QM}$ provides a maximum violation of the BCH inequality. For $\theta_1$, $\theta_2$, $\theta_1'$, $\theta_2'$ equal to 135°, 0°, 225°, and 90°, respectively, we find:

$$S_{\rm QM}(135^\circ, 0^\circ, 225^\circ, 90^\circ) = \frac{1}{2}\eta g \varepsilon_+ \left(1+\sqrt{2}F\left(\frac{\varepsilon_-}{\varepsilon_+}\right)^2\right) \quad (8)$$

In order to obtain the largest possible violation of the BCH inequality by the quantum mechanical prediction, the right hand side of Eq. (8) must exceed one, and the parameters $\eta$, $g$, $\varepsilon_M$, $F$ should be as close to unity as possible while $\varepsilon_m$ should be as small as possible.

Finally, any test of the Bell inequalities is statistical in nature; specifically, count rates have statistical errors and the measured quantities are ratios of coincidence count rates to singles count rates . These statistical errors must be sufficiently small so that the resulting error in the measured value of $S$ is much less than the magnitude of the violation (the amount by which $S_{\rm QM}$ exceeds one). Specifically, it is not the magnitude of the violation, but rather its comparison to the error limits of an experimentally determined value of $S$ that are a measure of the significance of the violation. As an aside it should be noted that some experimental parameters must be measured (and therefore have errors associated with them) in order to evaluate $S_{\rm QM}$; therefore, the quantum mechanical prediction $S_{\rm QM}$ will have errors associated with it.

## III. LOOPHOLES

Despite a substantial number of experimental tests of the Bell inequalities, no experiment to date has been entirely loophole free [10, 12, 17–20]. Specifically, one or more of the following loopholes were present: (1) the spatial correlation loophole (2) the detection efficiency loophole, and (3) the enforcement of locality (communication loophole).

**The first loophole -** the spatial correlation loophole - relates to the ideal case in which the two-particle entangled state is generated by a two-body decay that involves only the two particles that are entangled. In this case, momentum conservation ensures a strong spatial correlation that is necessary since the experimenter must make sure that both particles of an entangled pair end up in the detectors. This loophole is effectively eliminated by employing experiments based on two-body processes. The original Bell inequality experiments employing two photon atomic cascades suffered from spatial correlation. Specifically, the two entangled particles were the cascade photons but they were produced in the presence of a third particle, the atom, that carried off some momentum and smeared out the spatial correlation between the two photons.

**The second loophole -** the detection efficiency loophole - originates from experimental imperfections. Low detection efficiencies reduce the observable correlation, since the lower the detection efficiency, the lower the probability that both partners of an entangled pair will be detected. In particular, the Quantum mechanical expectation value $S_{QM}$ (Eq. 8) of the BCH inequality Eq. (5) involves the detection efficiency $\eta$. Clearly, if $\eta$ drops below some critical level the quantum mechanical prediction will always drop below one. Thus, a distinction between quantum mechanics and LHV theories is no longer possible. For the case of a symmetric entangled state as in Eq. (1) this critical value is $\eta_{\rm crit} = 0.82$ assuming all other parameters being perfect. However, it has been shown that for asymmetric entangled states (i.e. states for which the two components have weights different from $1/\sqrt{2}$) an efficiency of $\eta_{\rm crit} = 0.67$ would be sufficient [21]. The great benefit of atom based tests is that photo-ionization schemes can provide high detection efficiencies for almost all elements of the periodic table [22].

Since photon detectors had relatively low detection efficiencies, the CSHS and CH assumptions (cf. section I) were introduced in order to obtain experimentally testable inequalities. These assumptions allowed one to obtain inequalities that depend on ratios of coincidence rates to coincidence rates (e.g. Eq. 4) so that the detector efficiency did not appear in the quantum mechanical predictions for the inequality.

As pointed out by Santos [19] and as evident in the quantum mechanical prediction Eq. (8), high detection efficiency alone is not enough to test the BCH inequality. High detection efficiencies must also be accompanied by a high probability of actually detecting both particles of an entangled pair in their respective detectors, i.e. a high conditional probability $g$ of finding particle 2 in detector 2 provided that particle 1 entered detector 1. This also relates to the first loophole, spatial correlation.

**The third loophole -** enforcement of the locality condition - is also known as the communication loophole. It requires that the choice of correlation measurement for the two particles be completely independent at each analyzer. Specifically, one must guarantee no communication between the two analyzers during a measurement, i.e. the time interval from choices of analyzer "orientations" to detection must be outside each others space-time light cone.

This means that the times required for the two detection events must be short compared to the time required for a light signal to propagate from one analyzer/detector to the other. This involves not only a large separation between the two analyzer/detectors, but also the requirement for fast detection schemes and electronics. The time of a detection event as defined in this context is the time it takes to choose and set the analyser settings in a random fashion, analyze the spin state, and complete and record the outcome of the experiment.

The first attempt to enforce the locality condition was pursued by Aspect and coworkers [23]. However, their innovative experimental scheme did not allow a strict enforcement of the locality condition; in particular, they used periodic switching of the polarizer orientation rather than random orientations [18]. Very recently Zeilinger *et al.* did succeed in rigorously closing this loophole [24]. Zeilinger's success represents a very important step in the clarification of the EPR argument. However, since the experiment was performed with a detection efficiency of approximately 5%, the detection loophole is not yet closed. The ultimate definitive test must be one that enforces locality and simultaneously closes all other loopholes associated with previous tests.

## IV. ATOM BASED EXPERIMENTS

Significant progress in the experimental tests of Bell inequalities has been achieved. Most of the experiments have been performed using photons [23–33]. However, in this article we will concentrate on atom based experiments; these have features that make them distinctly different from their photon based counterparts [34]. For example, there are fermions as well as bosons, massive versus massless particles, long lived entangled states, and entanglement produced by direct manipulation of the atomic degrees of freedom. We will briefly review recent atom based tests of Bell inequalities and then conclude with a summary of the key ideas of the experiment which is underway at Texas A&M University.

### A. The Paris Experiments

In the Paris experiments, entanglement between atoms is produced using a high-Q micro-wave cavity, specifically, through the interaction of two completely independent Rydberg atoms with a common radiation field in the cavity. The two Rydberg atoms pass one after another through the cavity. With a proper choice of the interaction time between the atoms and the cavity field, entanglement between the two atoms is achieved [35].

While the detection efficiencies in this experiment are potentially high, the actual current implementation still has low detection efficiencies and relatively poor discrimination. Although these could be overcome, a principle difficulty for tests of the Bell inequalities based on entanglement in a micromaser is that the generation of the entanglement is a sequential process, which requires precise control of the experimental parameters such as interaction time of the atoms with the cavity fields, separation of atoms etc. The enforcement of Einstein locality would also be a problem since the detection of the atoms occurs sequentially.

### B. The Boulder Experiments

In the Boulder experiment, Wineland and coworkers [13] are the first group to close the detection efficiency loophole. They prepared two $Be^{+}$-ions in an ion trap in an entangled state between two Hyperfine Zeeman substates. The two substates can be identified with spin-up and spin-down and are therfore equivalent to an entangled state as depicted by Eq. 1.

The determination of the spin components is considerably more subtle than in the usual case of spin-1/2 particles. The measurement of a spin component with respect to a certain quantization axis is essentially equivalent to a classical manipulation of the spin, i.e. a rotation onto the quantization axis. The two possible results are parallel or anti-parallel, which then can be identified as spin-up or down with respect to the quantization axis. In the Boulder experiment, the classical manipulation occurs by irradiating the ion with two Raman beams, which prepare the ions in a "bright" and "dark" superposition of the two possible Zeeman substates. The phase of the Raman lasers determine the two phases. The actual measurement occurs by scattering photons off the ions. The "bright" state does scatter photons, while the "dark" state does not. The up-up or down-down outcome as defined in Eq. 3 corresponds to both ions bright, or both dark, respectively. Equivalently, an up-down or down-up outcome corresponds

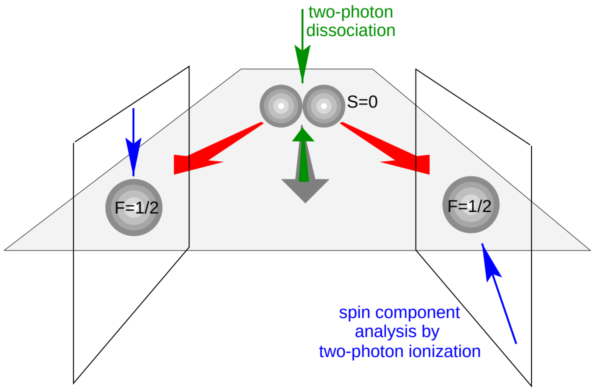


FIG. 1: Schematic of the Texas A&M experiment, which is a realization of Bohm's spin-1/2 particle EPR experiment. A $^{199}Hg_2$ molecule is dissociated into an entangled pair of atoms by a two-photon dissociation process. After the separation of the $^{199}$Hg atoms their spin components are measured by a two-photon ionization process.

to one ion dark and the other bright. By manipulation of the trap potential, the ions can be shifted within the trap without influencing the internal degrees of freedom, i.e. without affecting the entanglement. This shift in position then leads to a change in the relative phase of the Raman beams as seen by the two ions, and thereby enables the measurement of "spin" components in arbitrary directions.

Due to the proximity of the two ions in the trap, it is impossible in these experiments to enforce the locality condition. In fact, Vaidman has argued that the situation is even worse in these experiments [36]. Specifically, due to the detection of the state by the scattering of many photons, both detection events and the choice of both analyzer orientations overlap throughout a relatively long period of time (orders of magnitude longer than the time required for light to travel between the two ions).

### C. The Texas A&M Experiment

The need for a two-particle system in an entangled state in the test of Bell inequalities suggests that a diatomic molecule might be an ideal starting ground for such a test. In fact, assuming a nuclear spin 1/2 for each atom, the nuclear wave function of the molecule in the separated atom basis can be written as in Eq. 1. This state is identical to that of the two spin 1/2 particles in Bohm's classic version of the EPR *gedanken*experiment [9, 10]. The question is now whether it is possible to prepare a diatomic molecule in such a state. This is, in fact, possible, and in addition to our experiment, an approach using $Na_2$ dimer has been proposed by Shimony *et al.* [37]. Our experiment is essentially an implementation of Bohm's version of an EPR experiment (cf. fig. 1). For a more detailed discussion of the experiment, the reader is referred to the following set of references [15, 16, 38–42]. The experiment is based on $^{199}$Hg, which has nuclear spin I=1/2; it is the first attempt to actually test the BCH inequalities; (Eq. 5).

In our experiment $^{199}Hg_2$ dimers are generated in a supersonic jet expansion and are photo-dissociated using two laser beams in a stimulated Raman transition. The two wavelengths are at approximately 266 nm and 355 nm. Due to the difference in the photon energies, the molecule will be pumped onto a dissociating part of the potential energy surface of the ground electronic state. This leads to the dissociation of the molecule and thus to the spatial separation of the two atoms in the entangled state. The energy difference between a photon from each laser beam is distributed equally to the kinetic energies of the two atoms. Due to momentum conservation the atoms will fly apart in exactly opposite directions (180°) in the center-of-mass frame. However, as a consequence of the initial velocity of the dimers in the supersonic jet, the dissociated atoms will separate at angle of approximately 130° in the laboratory frame.

The preparation of the entangled state given by Eq. (1) requires selecting a molecular state with a total nuclear spin I=0. This is possible due to the specific symmetry rules for the total wavefunction of a homonuclear diatomic molecule consisting of two fermions. Thorough analysis of these symmetries [16] shows that only the entangled nuclear spin singlet state is dissociated if the excitation transition at 266 nm starts from states whose rotational quantum number N is even.

Determination of the correlation in the spin components of the two entangled atoms as well as detection of the atoms is achieved via a state-selective two-photon excitation-ionization scheme. Based on the selection rules for electronic transitions, one can photo-ionize only those atoms which are in the state $m_F = +1/2$, i.e. spin up along the quantization axis(cf. figure 3). Since the quantization axis is determined by the propagation direction of the left circular polarized laser beam, the spin component in any direction is measured by orienting the laser beam to propagate in that direction.

The ionized atoms are detected in two independent ways, i.e. by observing both the photo-ions and the photo-electrons. The photo-electrons are accelerated and guided by static electric fields onto a secondary electrode (CuBe surface), where they produce secondary electrons; the latter are then detected by a Channeltron multiplier. The photo-ions are focused, accelerated through 25 kV, and directed onto a slanted Al surface where they produce many secondary electrons that are also detected in a Channeltron multiplier.

Discussions of critical aspects of this experiment follow:

1. *purity of the state*: A high purity selection of the entangled state Eq. 1 to be dissociated is ensured since the different molecular rotational levels are spectroscopically well separated (typically >500 MHz). Furthermore the use of nuclear spins ensures that the entanglement is relatively immune to external influences such as stray magnetic fields. Specifically, since the nuclear magnetic moment is very small (approximately 1800 times smaller than the electron magnet moment), its interactions with the external environment are extremely weak. These factors lead to a very robust entangled state with very high purity.

2. *spatial correlation*: In order to be able to measure corre-

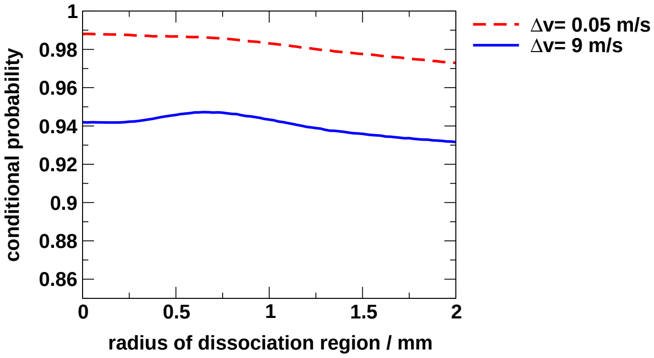


FIG. 2: Conditional probability $g$ as a function of the radius of the 1.5 mm long cylindrical dissociation region for two different velocity spreads of the dissociating dimers.

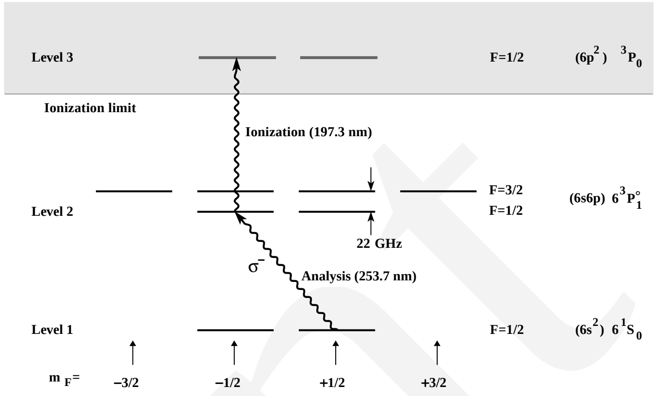


FIG. 3: Photo-ionization scheme used for spin analysis of the Hg atoms. The quantization axis is determined by the propagation direction of the left hand circular ($\sigma^-$) polarized 253.7 nm laser beam. The atom has been analyzed as "spin-up" ($m_F = +1/2$) if it is ionized in the two-photon process, 253.7 nm + 197.3 nm (via the $6^3P_1^\circ$ (F=1/2) state). Excitation/ionization transitions from $m_F = -1/2$ correspond to a "spin-down" analysis; by the selection rule for $\sigma^-$, $\Delta m_F = -1$, this transition must go via the $6^3P_1^\circ$ (F=3/2) state and is highly suppressed due to its 22 GHz detuning from the F=1/2 state.

lations with high efficiency, one must ensure that, with high probability, both of the entangled atoms from a dissociated dimer enter their respective analyzers. Since the photo dissociation process constitutes a two-body decay, the two atoms will move apart with a 180° separation in the center-of-mass (COM) frame; i.e. there is perfect spatial correlation in the COM. Consequently, if the particles are at rest, and are also generated in a point-like dissociation volume, the conditional probability $g$ defined in section II would be unity. However, in practice the spatial correlation is not perfect and $g$ is less than unity due to two factors. First, the dissociation volume is not a point; it is macroscopic and is formed by the common intersection of the supersonic jet and the two lasers. Second, the laboratory velocity of an atom is the vector addition of the dimer velocity with its COM velocity. Consequently, due to the small spread of the dimer velocities in the supersonic beam, there is a slight smearing of the spatial distribution of dissociated atom velocities in the laboratory frame. The smearing is significantly reduced by using the Doppler effect on the 266 nm transition to spectroscopically select and dissociate a narrow velocity subgroup from the initial dimer velocity distribution. Monte-Carlo simulations show that in our experiment values of $g$ greater than 0.94 can be expected based on the geometry of the beams and the velocity selection capabilities of our 266 nm laser system. This value of $g$ is dominated by the velocity spread. If the velocity spread in our system is reduced to $\Delta v$=0.05 m/s, the value of $g$ (greater than $\approx$ 0.98) is dominated by the size of the dissociation region, see figure 2.

3. *analyzer quality*: Two basic processes can reduce the quality of the spin analyzer. First, a Hg atom could be erroneously detected as spin up when it should have been detected as spin down, or vice-versa. Second, an atom is missed, i.e. it should have been detected as spin up, but no detection event is observed. The first error constitutes leakage, while the second is corresponds to non perfect transmission through the analyzers.

Theoretical analysis of the two-photon ionization process shows that, for a carefully chosen time separation between the analysis and ionization laser pulses, transmission in excess of 99% is possible. For 8 ns laser pulses with near-Fourier transform limited linewidths, this requires a relative time delay of 2-3 ns. This specification has been achieved using a Ti:Sapphire laser operating simultaneously at 761.1 nm (frequency tripled to yield 253.7 nm photons) and 789.9 nm (frequency quadrupled to yield 197.3 nm photons) [43]. Two factors can contribute to leakage in our analysis scheme, non-perfect polarization of the lasers, and excitation/ionization transitions through the $6^3P_1^\circ$ (F=3/2) state. Our simulations show that the latter is the most significant contribution to the leakage, and is on the order of $10^{-3}$. This leakage occurs despite the relatively large 22 GHz detuning of the $6^3P_1^\circ$ (F=3/2) state from the correct $6^3P_1^\circ$ (F=1/2) state.

4. *detector design and efficiency*: After the spin analysis and photo-ionization, the $Hg^+$ and/or its associated photo-electron must be detected with high efficiency and extremely low background. The latter is particularly important since the signal level must be reduced so that the probability of more than one Hg atom in the detection region at a time is negligible (otherwise coincidences between uncorrelated atoms from different dissociating molecules will be observed). Reducing photoelectron background signals to a negligible level is especially difficult in a vacuum system in the presence of ultraviolet laser beams that produce copious numbers of photoelectrons at walls and surfaces. Background signals associated with the ion detector have been easy to suppress to negligible levels.

Both the photo-ion and the photo-electron are observed with independent detectors. For confirmation of the detection of one of the atoms from a dissociated dimer, there are four choices. One can use the signal from the photo-ion detector alone; one can use the signal from the photo-electron detector alone; one can require a signal from both detectors (an AND configuration); or one can require a signal from either detector (an OR configuration). The AND configuration has the lowest overall detection efficiency but is least vulnerable to background signals. The OR configuration has the highest overall detection efficiency but is most vulnerable to background signals. In any case all four signals (an ion and electron detector signal for each atom of an entangled pair) are recorded and the inequality can be tested using each of the choices. This is an example of both the redundancy and the extensive systematic checks available with this experiment.

By observing both the photo-ion and the photo-electron, the measurement data inherently include measurements of the detection efficiencies, since the photo-electron and photo-ion are perfectly correlated. Knowing the detector efficiencies, one can use the measured coincidence rates for entangled atom pairs to also determine the conditional probability $g$ [42]. No *a priori* measurements of detector efficiencies are required; the same data used to test the Bell inequality can also be used to determine the efficiencies of the detectors while the data is being collected.

The acceptance solid angle of the detectors should be as large as possible, not only to maximize the signals (the usual situation), but also to maximize the conditional probability $g$.

Overall detection efficiencies close to unity for both the electron and the ion should be possible and have been confirmed by preliminary measurements [38].

5. *the locality loophole*: In our experiment the enforcement of Einstein locality, also known as the communication loophole, can be implemented by employing electro-optic modulators (EOM). Specifically, the EOM together with a beam splitting polarizer can, in a couple nanoseconds, change the propagation direction of the excitation laser beam and hence the component of nuclear spin angular momentum being observed. A separation between our detectors of approximately 12 m will be necessary in order to close the locality loophole. This estimate includes allowances for the selection of a random number, switching the EOM, firing the nanosecond-detection lasers, the run-time of the electrons through the detection system including the Channeltron, as well as digitizing the detector output.

## V. SUMMARY

John Bell's work on the foundations of Quantum Mechanics continues to influence and inspire Physics. The quest for a final answer in the test of Bell inequalities and thus the answer to the question whether or not quantum mechanics is a complete theory creates new ideas for more and more refined experiments. Simultaneously closing the existing loopholes has proven difficult. However, the atom based experiments that have emerged in the last few years are especially promising.

## VI. ACKNOWLEDGMENTS

This research was supported by the Robert A. Welch Foundation grant No. A-1218, the Texas Advanced Research Program grant 010366-0367-1999, and by the National Science Foundation grant PHY-9732459. The authors thank John Clauser and Marlan Scully for many helpful discussions on the theory and encouragement as well as Joel Bondurant, Robert Kenefick, Cechan Tian and Seiichirou Yokoo for their experimental contributions and active participation.

## REFERENCES


[1] A. Steane, Rep. Prog. Phys. **61**, 117 (1998).
[2] P. Rodgers, in *Physics World (special issue)* (1998), vol. 11, pp. 33–57.
[3] W. Buttler, R. Hughes, P. Kwiat, S. Lamoreaux, G. Luher, G. Morgan, J. Nordjolt, C. Peterson, and C. Simmons, Phys. Rev. Lett. **81**, 3283 (1998).
[4] D. Bouwmeester, J.-W. Pan, K. Mattle, M. Eible, H. Weinfurter, and A. Zeilinger, Nature **390**, 575 (1997).
[5] D. Boschi, S. Branca, F. DeMartini, L. Hardy, and S. Popescu, Phys. Rev. Lett. **80**, 1121 (1998).
[6] A. Furusawa, J. Sørensen, S. Braunstein, C. Fuchs, H. Kimble, and E. Polziak, Science **282**, 706 (1998).
[7] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935).
[8] J. Bell, Physics **1**, 195 (1964), reprinted in J.S. Bell, (1987), *Speakable and Unspeakable in Quantum Mechanics*, Cambridge University Press.
[9] D. Bohm, *Quantum Physics* (Prentice Hall, New York, 1951).
[10] J. Clauser and A. Shimony, Rep. Prog. Phys **41**, 1881 (1978).
[11] J. Clauser, M. Horne, A. Shimony, and R. Holt, Phys. Rev. Lett. **23**, 880 (1969).
[12] J. Clauser and M. Horne, Phys. Rev. D **10**, 526 (1974).
[13] M. Rowe, D. Kielpinski, V. Meyer, C. Sackett, W. Itano, C. Monroe, and D. Wineland, Nature **409**, 791 (2001).
[14] S. Freedman, Rep. No. LBL 191, Lawrence Livermore Lab. (1972).
[15] E. Fry, T. Walther, and S. Li, Phys. Rev. A **52**, 4381 (1995).
[16] T. Walther and E. Fry, Zeitschr. f. Naturforschung **52a**, 20 (1997), Proceedings of the workshop in honor of E.C.G. Sudarshan.
[17] P. Pearle, Phys. Rev. D **2**, 1418 (1970).
[18] A. Zeilinger, Phys. Lett. A **118**(1), 1 (1986).

[19] E. Santos, Phys. Lett. A **212**, 10 (1996).
[20] N. Gisin and B. Gisin, Phys. Lett. A **260**, 323 (1999).
[21] P. Eberhard, Phys. Rev. A **47**, 747 (1993).
[22] G. Hurst, M. Payne, S. Kramer, and J. Young, Rev. of Mod. Phys. **51**, 767 (1979).
[23] A. Aspect, J. Dalibard, and G. Roger, Phys. Rev. Lett. **49**, 1804 (1982).
[24] G. Weihs, T. Jennewein, C. Simon, H. Weinfurter, and A. Zeilinger, Phys. Rev. **81**(23), 5039 (1998).
[25] S. Freedman and J. Clauser, Phys. Rev. Lett. **28**, 938 (1972).
[26] J. Clauser, Phys. Rev. Lett. **36**, 1223 (1976).
[27] E. Fry and R. Thompson, Phys. Rev. Lett. **37**, 465 (1976).
[28] A. Aspect, P. Grangier, and G. Roger, Phys. Rev. Lett. **49**, 91 (1982).
[29] C. Alley and Y. H. Shih, in *Proc. Int. Symp. Foundations of Quantum Mechanics*, edited by M. Namiki (1987), p. 47.
[30] Z. Ou and L. Mandel, Phys. Rev. Lett. **61**, 50 (1988).
[31] P. Kwiat, K. Mattle, H. Weinfurter, A. Zeilinger, A. Sergienko, and Y. Shih, Phys. Rev. Lett. **75**, 4337 (1995).
[32] P. Kwiat, E. Waks, A. White, I. Appelbaum, and P. Eberhard, quant-ph/9810003 (1998).
[33] W. Tittel, J. Brendel, H. Zbinden, and N. Gisin, Phys. Rev. Lett. **81**, 3563 (1998).
[34] R. Nicolaescu, T. Walther, E. S. Fry, and M. Muendel, Appl. Optics **38**, 1784 (1999).
[35] E. Hagley, X. Maître, G. Nogues, C. Wunderlich, M. Brune, J. Raimond, and S. Haroche, Phys. Rev. **79**, 1 (1997).
[36] L. Vaidman, quant-ph/0102139 (2001).
[37] T. Lo and A. Shimony, Phys. Rev. A **23**, 3003 (1981).
[38] E. Fry, T. Walther, and R. Kenefick, Physica Scripta **T76**, 47 (1998).
[39] E. Fry and T. Walther, in *Experimental Metaphysics - Quantum Mechanical Studies for Abner Shimony, Vol. I*, edited by R. Cohen and J. Stachel (Kluwer Academic, Dordrecht, 1996).
[40] E. Fry and T. Walther, in *Quantum Interferometry*, edited by F. D. Martini, G. Denardo, and Y. Shih (VCH Publishing group, Weinheim, 1996).
[41] E. Fry and T. Walther, Adv. At. Mol. Opt. Phys. **42B**, 1 (1999).
[42] T. Walther and E. Fry, in *New Developments on Fundamental Problems in Quantum Physics*, edited by M. Ferrero, E. Santos, and S. Huelga (Kluwer Academic, Dordrecht, 1997), vol. 81 of *Fundamental Theories of Physics*.
[43] C. Tian, T. Walther, R. Nicolaescu, X. Pan, Y. Liao, and E. Fry, Opt. Lett. **24**, 1496 (1999).