Example#1, 2-loop planar vertex with three different masses in d=6-2*eps

```
<< AMBREv1.3.m
```

```
by K.Kajda   ver: 1.3
last modified Jul 2016
last executed on 25.07.2016 at 17:27
```

```
<< PlanarityTestv1.2.m
```

```
by E. Dubovyk and K. Bielas ver: 1.2
created: January 2014
last executed: 25.07.2016 at 17:27
```

```
invariants = {p1^2 → 0, p2^2 → 0, p1 * p2 → s / 2};
```

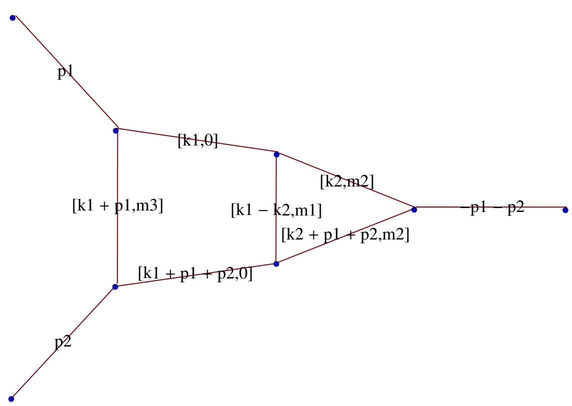
```
d = 6 - 2 eps; (* by default d=4-2 eps *)
```

```
ex = PR[k1, 0, n1] PR[k1 - k2, m1, n2] PR[k2, m2, n3]
   PR[k1 + p1, m3, n4] PR[k1 + p1 + p2, 0, n5] PR[k2 + p1 + p2, m2, n6];
```

```
PlanarityTest[{ex}, {k1, k2}, DrawGraph → True];
```

```
The Diagram
```

```
is planar.
```




```
Fullintegral[{1}, {ex}, {k2, k1}];
```

```
IntPart[1]
```

```
numerator=1
integral=PR[k1 - k2, m1, n2] PR[k2, m2, n3] PR[k2 + p1 + p2, m2, n6]
momentum=k2
```

Fauto::mode :
U and F polynomials will be calculated in AUTO mode. In order to use MANUAL mode execute Fauto[0].

```
SubLoop[integral]
```

```
Iteration nr1: >>Integrating over k2<<

Computing U & F polynomial in AUTO mode >>Fauto[1]<<

U polynomial...
X[1] + X[2] + X[3]
F polynomial...
```

$m1^2 X[1] + m2^2 X[2] - PR[k1, 0] X[1] X[2] + m2^2 X[3] - PR[k1 + p1 + p2, 0] X[1] X[3] - s X[2] X[3]$

```
Representation after integrating over: k2...
```

$$\text{SubLoop1}\Big[\Big((-1)^{\frac{1}{2}(6-2\,eps)+\frac{1}{2}(-6+2\,eps)+n2+n3+n6+z3+z5}\,(m1^2)^{z1}\,(m2^2)^{z2+z4}\,(-s)^{\frac{1}{2}(6-2\,eps)-n2-n3-n6-z1-z2-z3-z4-z5}$$
$$\text{Gamma}[-z1]\,\text{Gamma}[-z2]\,\text{Gamma}\Big[\frac{1}{2}(6-2\,eps)-n2-n3-z1-z2-z3\Big]$$
$$\text{Gamma}[-z3]\,\text{Gamma}[-z4]\,\text{Gamma}\Big[\frac{1}{2}(6-2\,eps)-n2-n6-z1-z4-z5\Big]\,\text{Gamma}[-z5]$$
$$\text{Gamma}[n2+z1+z3+z5]\,\text{Gamma}\Big[\frac{1}{2}(-6+2\,eps)+n2+n3+n6+z1+z2+z3+z4+z5\Big]\Big)\Big/$$
$$(\text{Gamma}[n2]\,\text{Gamma}[n3]\,\text{Gamma}[n6]\,\text{Gamma}[6-2\,eps-n2-n3-n6-z1-z2-z4]),$$
$$(-1)^{\frac{1}{2}(6-2\,eps)+\frac{1}{2}(-6+2\,eps)}\,s^{\frac{1}{2}(6-2\,eps)+\frac{1}{2}(-6+2\,eps)}\,PR[k1, 0, z3]\,PR[k1+p1+p2, 0, z5]\Big]$$

IntPart[2]

```
numerator=1
integral=PR[k1, 0, n1 - z3] PR[k1 + p1, m3, n4] PR[k1 + p1 + p2, 0, n5 - z5]
momentum=k1
```

Fauto::mode :
U and F polynomials will be calculated in AUTO mode. In order to use MANUAL mode execute Fauto[0].

repr = SubLoop[integral]

```
Iteration nr2: >>Integrating over k1<<

Computing U & F polynomial in AUTO mode >>Fauto[1]<<

U polynomial...
```

$X[1] + X[2] + X[3]$

```
F polynomial...
```

$m3^2\, X[2] - s\, X[1]\, X[3]$

```
Final representation:
```

$$\Big((-1)^{n1+n2+n3+n4+n5+n6} \left(m1^2\right)^{z1} \left(m2^2\right)^{z2+z4} \left(m3^2\right)^{z6} (-s)^{6-2\,eps-n1-n2-n3-n4-n5-n6-z1-z2-z4-z6}$$
$$\text{Gamma}[-z1]\,\text{Gamma}[-z2]\,\text{Gamma}\Big[\frac{1}{2}(6-2\,eps)-n2-n3-z1-z2-z3\Big]$$
$$\text{Gamma}[-z3]\,\text{Gamma}[-z4]\,\text{Gamma}\Big[\frac{1}{2}(6-2\,eps)-n2-n6-z1-z4-z5\Big]\,\text{Gamma}[-z5]$$
$$\text{Gamma}[n2+z1+z3+z5]\,\text{Gamma}\Big[\frac{1}{2}(-6+2\,eps)+n2+n3+n6+z1+z2+z3+z4+z5\Big]$$
$$\text{Gamma}\Big[\frac{1}{2}(6-2\,eps)-n1-n4+z3-z6\Big]\,\text{Gamma}\Big[\frac{1}{2}(6-2\,eps)-n4-n5+z5-z6\Big]$$
$$\text{Gamma}[-z6]\,\text{Gamma}[n4+z6]\,\text{Gamma}\Big[\frac{1}{2}(-6+2\,eps)+n1+n4+n5-z3-z5+z6\Big]\Big)\Big/$$
$$(\text{Gamma}[n2]\,\text{Gamma}[n3]\,\text{Gamma}[n4]\,\text{Gamma}[n6]\,\text{Gamma}[n1-z3]\,\text{Gamma}[6-2\,eps-n2-n3-n6-z1-z2-z4]\,\text{Gamma}[n5-z5]\,\text{Gamma}[6-2\,eps-n1-n4-n5+z3+z5-z6])$$

$$\Big((-1)^{n1+n2+n3+n4+n5+n6} \left(m1^2\right)^{z1} \left(m2^2\right)^{z2+z4} \left(m3^2\right)^{z6} (-s)^{6-2\,eps-n1-n2-n3-n4-n5-n6-z1-z2-z4-z6}$$
$$\text{Gamma}[-z1]\,\text{Gamma}[-z2]\,\text{Gamma}\Big[\frac{1}{2}(6-2\,eps)-n2-n3-z1-z2-z3\Big]$$
$$\text{Gamma}[-z3]\,\text{Gamma}[-z4]\,\text{Gamma}\Big[\frac{1}{2}(6-2\,eps)-n2-n6-z1-z4-z5\Big]\,\text{Gamma}[-z5]$$
$$\text{Gamma}[n2+z1+z3+z5]\,\text{Gamma}\Big[\frac{1}{2}(-6+2\,eps)+n2+n3+n6+z1+z2+z3+z4+z5\Big]$$
$$\text{Gamma}\Big[\frac{1}{2}(6-2\,eps)-n1-n4+z3-z6\Big]\,\text{Gamma}\Big[\frac{1}{2}(6-2\,eps)-n4-n5+z5-z6\Big]$$
$$\text{Gamma}[-z6]\,\text{Gamma}[n4+z6]\,\text{Gamma}\Big[\frac{1}{2}(-6+2\,eps)+n1+n4+n5-z3-z5+z6\Big]\Big)\Big/$$
$$(\text{Gamma}[n2]\,\text{Gamma}[n3]\,\text{Gamma}[n4]\,\text{Gamma}[n6]\,\text{Gamma}[n1-z3]\,\text{Gamma}[6-2\,eps-n2-n3-n6-z1-z2-z4]\,\text{Gamma}[n5-z5]\,\text{Gamma}[6-2\,eps-n1-n4-n5+z3+z5-z6])$$


```
fin = BarnesLemma[#, 1, Shifts → True] & /@ {repr} // Simplify
```

```
>> Shifting: {z2 → z2 - z4}

>> Barnes 1st Lemma will be checked for: {z5, z4, z3} <<
   Starting with dim=6 representation...

1. Checking z5
2. Checking z4...Barnes Lemma was applied.
3. Checking z3

>> Representation after 1st Barnes Lemma:  <<

   1st Barnes Lemma was applied for: {z4}
   Obtained representation has: dim=5
```

$$\Big\{\Big((-1)^{n1+n2+n3+n4+n5+n6}\,(m1^2)^{z1}\,(m2^2)^{z2}\,(m3^2)^{z6}\,(-s)^{6-2\,eps-n1-n2-n3-n4-n5-n6-z1-z2-z6}$$

```
Gamma[-z1] Gamma[-z2] Gamma[3 - eps - n2 - n3 - z1 - z2 - z3] Gamma[-z3]
Gamma[3 - eps - n2 - n6 - z1 - z2 - z5] Gamma[6 - 2 eps - 2 n2 - n3 - n6 - 2 z1 - z2 - z3 - z5]
Gamma[-z5] Gamma[n2 + z1 + z3 + z5] Gamma[-3 + eps + n2 + n3 + n6 + z1 + z2 + z3 + z5]
Gamma[3 - eps - n1 - n4 + z3 - z6] Gamma[3 - eps - n4 - n5 + z5 - z6]
Gamma[-z6] Gamma[n4 + z6] Gamma[-3 + eps + n1 + n4 + n5 - z3 - z5 + z6]) /
(Gamma[n2] Gamma[n3] Gamma[n4] Gamma[n6] Gamma[6 - 2 eps - n2 - n3 - n6 - z1 - z2]
Gamma[n1 - z3] Gamma[n5 - z5] Gamma[6 - 2 eps - 2 n2 - n3 - n6 - 2 z1 - 2 z2 - z3 - z5]
Gamma[6 - 2 eps - n1 - n4 - n5 + z3 + z5 - z6])}
```

finres = fin /. {n1 → 1, n2 → 1, n3 → 1, n4 → 1, n5 → 1, n6 → 1}

$$\Big\{\Big((m1^2)^{z1}\,(m2^2)^{z2}\,(m3^2)^{z6}\,(-s)^{-2\,eps-z1-z2-z6}$$

```
Gamma[-z1] Gamma[-z2] Gamma[1 - eps - z1 - z2 - z3] Gamma[-z3]
Gamma[1 - eps - z1 - z2 - z5] Gamma[2 - 2 eps - 2 z1 - z2 - z3 - z5] Gamma[-z5]
Gamma[1 + z1 + z3 + z5] Gamma[eps + z1 + z2 + z3 + z5] Gamma[1 - eps + z3 - z6]
Gamma[1 - eps + z5 - z6] Gamma[-z6] Gamma[1 + z6] Gamma[eps - z3 - z5 + z6]) /
(Gamma[3 - 2 eps - z1 - z2] Gamma[1 - z3] Gamma[1 - z5]
Gamma[2 - 2 eps - 2 z1 - 2 z2 - z3 - z5] Gamma[3 - 2 eps + z3 + z5 - z6])}
```

<< MB.m

```
MB 1.2

by Michal Czakon

improvements by Alexander Smirnov

more info in hep-ph/0511200

last modified 2 Jan 09
```

<< MBresolve.m

```
MBresolve 1.0

by Alexander Smirnov

more info in arXiv:0901.0386

last modified 4 Jan 09
```

```
step1 = MBresolve[#, eps] & /@ finres // Flatten;
```

```
CREATING RESIDUES LIST..........1.2273 seconds
EVALUATING RESIDUES..........0.0325 seconds
```

```
step2 = MBexpand[step1, Exp[2 * eps * EulerGamma], {eps, 0, 0}];
MBanalytic = MBmerge[step2]
```

```
{MBint[1/(32 eps^2) (4 + 18 eps + eps^2 (53 + 2 π^2) - 4 eps (2 + 9 eps) Log[m3^2] + 8 eps^2 Log[m3^2]^2),
  {{eps → 0}, {}}], MBint[1/(16 eps) Gamma[1 - z5] Gamma[-z5] Gamma[z5] Gamma[1 + z5]
   (2 + 9 eps - 4 eps Log[m3^2] - 2 eps PolyGamma[0, 1 - z5] + 2 eps PolyGamma[0, z5]),
  {{eps → 0}, {z5 → 0.85806}}],
 MBint[1/(2 eps Gamma[3 - z6]) (m3^2)^z6 (-s)^-z6 Gamma[1 - z6]^2 Gamma[-z6] Gamma[z6]
    Gamma[1 + z6] (1 + 3 eps + 2 eps EulerGamma - 2 eps Log[-s] - 3 eps PolyGamma[0, 1 - z6] +
      3 eps PolyGamma[0, 3 - z6] + 2 eps PolyGamma[0, z6]),
  {{eps → 0}, {z6 → -0.224153}}], MBint[((m3^2)^z6 (-s)^-z6 Gamma[-z5] Gamma[z5]
     Gamma[1 - z5 - z6] Gamma[1 + z5 - z6] Gamma[-z6] Gamma[z6] Gamma[1 + z6]) /
   (2 Gamma[3 - z6]), {{eps → 0}, {z5 → 0.140245, z6 → -0.179148}}],
 MBint[((m2^2)^(-z3-z5) (m3^2)^z6 (-s)^(z3+z5-z6) Gamma[-z3] Gamma[1 + z3] Gamma[-z5]
     Gamma[1 + z5] Gamma[z3 + z5] Gamma[1 + z3 + z5] Gamma[1 + z3 - z6]
     Gamma[1 + z5 - z6] Gamma[-z6] Gamma[1 + z6] Gamma[-z3 - z5 + z6]) /
   (Gamma[1 - z3] Gamma[1 - z5] Gamma[2 + z3 + z5] Gamma[3 + z3 + z5] Gamma[3 + z3 + z5 - z6]),
  {{eps → 0}, {z3 → -0.276442, z5 → -0.459608, z6 → -0.446372}}], MBint[
  ((m2^2)^z2 (m3^2)^z6 (-s)^(-z2-z6) Gamma[-z2] Gamma[1 - z2 - z3] Gamma[-z3] Gamma[1 - z2 - z5]
     Gamma[2 - z2 - z3 - z5] Gamma[-z5] Gamma[1 + z3 + z5] Gamma[z2 + z3 + z5]
     Gamma[1 + z3 - z6] Gamma[1 + z5 - z6] Gamma[-z6] Gamma[1 + z6] Gamma[-z3 - z5 + z6]) /
   (Gamma[3 - z2] Gamma[1 - z3] Gamma[1 - z5] Gamma[2 - 2 z2 - z3 - z5]
     Gamma[3 + z3 + z5 - z6]),
  {{eps → 0}, {z2 → -0.200233, z3 → -0.101036, z5 → -0.483143, z6 → -0.25871}}],
 MBint[((m1^2)^z1 (m2^2)^z2 (m3^2)^z6 (-s)^(-z1-z2-z6) Gamma[-z1] Gamma[-z2] Gamma[1 - z1 - z2 - z3]
     Gamma[-z3] Gamma[1 - z1 - z2 - z5] Gamma[2 - 2 z1 - z2 - z3 - z5] Gamma[-z5]
     Gamma[1 + z1 + z3 + z5] Gamma[z1 + z2 + z3 + z5] Gamma[1 + z3 - z6] Gamma[1 + z5 - z6]
     Gamma[-z6] Gamma[1 + z6] Gamma[-z3 - z5 + z6]) / (Gamma[3 - z1 - z2]
     Gamma[1 - z3] Gamma[1 - z5] Gamma[2 - 2 z1 - 2 z2 - z3 - z5] Gamma[3 + z3 + z5 - z6]),
  {{eps → 0}, {z1 → 0.507578, z2 → -0.272769, z3 → -0.0825234,
    z5 → -0.0730761, z6 → -0.0666452}}]}
```

```
MBintegrate[MBanalytic, {s → -1, m1 → 1, m2 → 2, m3 → 3}]
```

```
Shifting contours...

Performing 4 lower-dimensional integrations with NIntegrate...1...2...3...4

Higher-dimensional integrals

Preparing MBpart1eps0 (dim 5)

Preparing MBpart2eps0 (dim 4)

Preparing MBpart3eps0 (dim 3)

Preparing MBpart4eps0 (dim 2)

Running MBpart1eps0

Running MBpart2eps0

Running MBpart3eps0

Running MBpart4eps0
```

$$\left\{0.216796 + \frac{0.125}{\text{eps}^2} - \frac{0.141395}{\text{eps}}, \{0.000274603, 0\}\right\}$$

```
Quit[]

<< ../FIESTA3.2/FIESTA3.m

UsingQLink = False; UsingC = False;

d0 = 6;
```

```
SDEvaluate[UF[{k1, k2}, {-k1^2, -(k1 - k2)^2 + m1^2, -k2^2 + m2^2, -(k1 + p1)^2 + m3^2,
   -(k1 + p1 + p2)^2, -(k2 + p1 + p2)^2 + m2^2}, {p1^2 → 0, p2^2 → 0, p1 * p2 → s / 2,
   s → -1, m1 → 1, m2 → 2, m3 → 3}], {1, 1, 1, 1, 1, 1}, 0]
```

```
FIESTA 3.2
Starting 1 subkernels
Subkernel will be used for launching external programs, all evaluations go on main kernel.
UsingC: False
NumberOfLinks: 1
UsingQLink: False
Strategy: STRATEGY_S
Integration has to be performed up to order 1
Sector decomposition - 6 sectors
Primary sector 1 resulted in 9 sectors.
Primary sector 2 resulted in 5 sectors.
Primary sector 3 resulted in 4 sectors.
Primary sector 4 resulted in 3 sectors.
Primary sector 5 resulted in 9 sectors.
Primary sector 6 resulted in 4 sectors.
Totally: 0.2702 seconds; 12 sectors.
Preparing database: 0.0009 seconds.
Variable substitution..........0.164 seconds; 34 terms.
Pole resolution..........0.0376 seconds; 49 terms.
Expression preparation..........0.0633 seconds; 49 terms.
Epsilon expansion..........0.0536 seconds; 113 terms.
Preparing integration strings..........0.117 seconds; 113 terms.
Database ready for integration.
Terms of order -1: 15, max vars: 4
Integrating.........5.9445 seconds.
Returned answer: 0.2500100709026885 + pm* 0.00002228915082049683
(0.12500503545134428 + 0.000011144575410248419*pm1)*ep^(-2)
Terms of order 0: 49, max vars: 5
Integrating.........16.1048 seconds.
Returned answer: -0.28287657366166613 + pm* 0
(0.12500503545134428 + 0.000011144575410248419*pm2)*ep^(-2)+(-0.1414382868308331)*ep^(-1)
Terms of order 1: 49, max vars: 5
Integrating.........20.4259 seconds.
Returned answer: -0.3893586055083057 + pm* 0
(0.12500503545134428 + 0.000011144575410248419*pm4)*ep^(-2)+(-0.1414382868308331)*ep^(-1)+
Total integration time: 42.4838
Total time used: 43.3877 seconds.
```

$$0.216571 - \frac{0.141438}{ep} + \frac{0.125005 + 0.0000111446\,pm7}{ep^2} + 0.0000366642\,pm9$$

```
Quit[]
```

The same diagram as above. To get the same dimensionality one needs to modify F by hand on each step.

```
<< AMBREv1.2.m
```

```
by K.Kajda   ver: 1.2
last modified 9 Apr 2008
last executed on 25.07.2016 at 17:46
```

```
invariants = {p1^2 → 0, p2^2 → 0, p1 * p2 → s / 2};
```

```
ex = PR[k1, 0, n1] PR[k1 - k2, m1, n2] PR[k2, m2, n3]
   PR[k1 + p1, m3, n4] PR[k1 + p1 + p2, 0, n5] PR[k2 + p1 + p2, m2, n6];
```

```
Fullintegral[{1}, {ex}, {k2, k1}];

IntPart[1]

numerator=1
integral=PR[k1 - k2, m1, n2] PR[k2, m2, n3] PR[k2 + p1 + p2, m2, n6]
momentum=k2
```

Fauto::mode :
U and F polynomials will be calculated in AUTO mode. In order to use MANUAL mode execute Fauto[0].

```
SubLoop[integral]

Iteration nr1: >>Integrating over k2<<

Computing U & F polynomial in AUTO mode >>Fauto[1]<<

U polynomial...
X[1] + X[2] + X[3]
F polynomial...
m2^2 FX[X[2] + X[3]]^2 + m1^2 X[1]^2 + m1^2 X[1] X[2] + m2^2 X[1] X[2] - PR[k1, 0] X[1] X[2] +
  m1^2 X[1] X[3] + m2^2 X[1] X[3] - PR[k1 + p1 + p2, 0] X[1] X[3] - s X[2] X[3]

Representation after integrating over: k2...

SubLoop1[
 ((-1)^(n2+n3+n6+z5+z8) (m1^2)^(z2+z3+z6) (m2^2)^(z1+z4+z7) (-s)^(2-eps-n2-n3-n6-z1-z2-z3-z4-z5-z6-z7-z8) Gamma[-z1]
    Gamma[-z2] Gamma[-z3] Gamma[-z4] Gamma[-z5] Gamma[-z6] Gamma[-z7]
    Gamma[-z8] Gamma[-2 + eps + n2 + n3 + n6 + z1 + z2 + z3 + z4 + z5 + z6 + z7 + z8] Gamma[
     n2 + 2 z2 + z3 + z4 + z5 + z6 + z7 + z8] Gamma[2 - eps - n2 - n3 + z1 - z2 - z3 - z4 - z5 - z9]
    Gamma[-z9] Gamma[-2 z1 + z9] Gamma[2 - eps - n2 - n6 - z1 - z2 - z6 - z7 - z8 + z9]) /
  (Gamma[n2] Gamma[n3] Gamma[4 - 2 eps - n2 - n3 - n6] Gamma[n6] Gamma[-2 z1]),
 PR[k1, 0, z5] PR[k1 + p1 + p2, 0, z8]]

Fauto[0]
```

Fmanual::mode : U and F polynomials will be calculated in MANUAL mode. Now you can modify F polynomial (fupc).

```
U polynomial...
X[1] + X[2] + X[3]
F polynomial...
m2^2 FX[X[2] + X[3]]^2 + m1^2 X[1]^2 + m1^2 X[1] X[2] + m2^2 X[1] X[2] - PR[k1, 0] X[1] X[2] +
  m1^2 X[1] X[3] + m2^2 X[1] X[3] - PR[k1 + p1 + p2, 0] X[1] X[3] - s X[2] X[3]

fupc = m2^2 FX[X[2] + X[3]]^2 + m1^2 X[1]^2 + m1^2 X[1] X[2] + m2^2 X[1] X[2] - PR[k1, 0] X[1] X[2] +
  m1^2 X[1] X[3] + m2^2 X[1] X[3] - PR[k1 + p1 + p2, 0] X[1] X[3] - s X[2] X[3]

fupc =m1^2 X[1] + m2^2 X[2] - PR[k1, 0] X[1] X[2] +
  m2^2 X[3] - PR[k1 + p1 + p2, 0] X[1] X[3] - s X[2] X[3]

m1^2 X[1] + m2^2 X[2] - PR[k1, 0] X[1] X[2] + m2^2 X[3] - PR[k1 + p1 + p2, 0] X[1] X[3] - s X[2] X[3]

SubLoop[integral]
```

```
Iteration nr1: >>Integrating over k2<<

U & F polynomial was computed by user >>Fauto[0]<<

Representation after integrating over: k2...

SubLoop1[
  ((-1)^(n2+n3+n6+z3+z5) (m1^2)^z1 (m2^2)^(z2+z4) (-s)^(2-eps-n2-n3-n6-z1-z2-z3-z4-z5) Gamma[-z1] Gamma[-z2]
     Gamma[2 - eps - n2 - n3 - z1 - z2 - z3] Gamma[-z3] Gamma[-z4]
     Gamma[2 - eps - n2 - n6 - z1 - z4 - z5] Gamma[-z5] Gamma[n2 + z1 + z3 + z5]
     Gamma[-2 + eps + n2 + n3 + n6 + z1 + z2 + z3 + z4 + z5]) /
   (Gamma[n2] Gamma[n3] Gamma[n6] Gamma[4 - 2 eps - n2 - n3 - n6 - z1 - z2 - z4]),
  PR[k1, 0, z3] PR[k1 + p1 + p2, 0, z5]]
```

IntPart[2]

```
numerator=1
integral=PR[k1, 0, n1 - z3] PR[k1 + p1, m3, n4] PR[k1 + p1 + p2, 0, n5 - z5]
momentum=k1
```

Fauto::mode :
U and F polynomials will be calculated in AUTO mode. In order to use MANUAL mode execute Fauto[0].

Fauto[0]

Fmanual::mode : U and F polynomials will be calculated in MANUAL mode. Now you can modify F polynomial (fupc).

```
U polynomial...
X[1] + X[2] + X[3]
F polynomial...
m3^2 X[1] X[2] + m3^2 X[2]^2 - s X[1] X[3] + m3^2 X[2] X[3]

fupc = m3^2 X[1] X[2] + m3^2 X[2]^2 - s X[1] X[3] + m3^2 X[2] X[3]
```

fupc = m3^2 X[2] - s X[1] X[3]

```
m3^2 X[2] - s X[1] X[3]
```

repr = SubLoop[integral]

```
Iteration nr2: >>Integrating over k1<<

U & F polynomial was computed by user >>Fauto[0]<<

Final representation:
```

$$\Big((-1)^{n1+n2+n3+n4+n5+n6}\,(m1^2)^{z1}\,(m2^2)^{z2+z4}\,(m3^2)^{z6}\,(-s)^{4-2\,eps-n1-n2-n3-n4-n5-n6-z1-z2-z4-z6}$$

```
    Gamma[-z1] Gamma[-z2] Gamma[2 - eps - n2 - n3 - z1 - z2 - z3]
    Gamma[-z3] Gamma[-z4] Gamma[2 - eps - n2 - n6 - z1 - z4 - z5] Gamma[-z5]
    Gamma[n2 + z1 + z3 + z5] Gamma[-2 + eps + n2 + n3 + n6 + z1 + z2 + z3 + z4 + z5]
    Gamma[2 - eps - n1 - n4 + z3 - z6] Gamma[2 - eps - n4 - n5 + z5 - z6]
    Gamma[-z6] Gamma[n4 + z6] Gamma[-2 + eps + n1 + n4 + n5 - z3 - z5 + z6]) /
  (Gamma[n2] Gamma[n3] Gamma[n4] Gamma[n6] Gamma[n1 - z3] Gamma[
     4 - 2 eps - n2 - n3 - n6 - z1 - z2 - z4] Gamma[n5 - z5] Gamma[4 - 2 eps - n1 - n4 - n5 + z3 + z5 - z6])
```

$$\Big((-1)^{n1+n2+n3+n4+n5+n6}\,(m1^2)^{z1}\,(m2^2)^{z2+z4}\,(m3^2)^{z6}\,(-s)^{4-2\,eps-n1-n2-n3-n4-n5-n6-z1-z2-z4-z6}$$

```
    Gamma[-z1] Gamma[-z2] Gamma[2 - eps - n2 - n3 - z1 - z2 - z3]
    Gamma[-z3] Gamma[-z4] Gamma[2 - eps - n2 - n6 - z1 - z4 - z5] Gamma[-z5]
    Gamma[n2 + z1 + z3 + z5] Gamma[-2 + eps + n2 + n3 + n6 + z1 + z2 + z3 + z4 + z5]
    Gamma[2 - eps - n1 - n4 + z3 - z6] Gamma[2 - eps - n4 - n5 + z5 - z6]
    Gamma[-z6] Gamma[n4 + z6] Gamma[-2 + eps + n1 + n4 + n5 - z3 - z5 + z6]) /
  (Gamma[n2] Gamma[n3] Gamma[n4] Gamma[n6] Gamma[n1 - z3]
    Gamma[4 - 2 eps - n2 - n3 - n6 - z1 - z2 - z4] Gamma[n5 - z5]
    Gamma[4 - 2 eps - n1 - n4 - n5 + z3 + z5 - z6])
```

fin = BarnesLemma[#, 1, Shifts → True] & /@ {repr} // Simplify

```
>> Shifting: {z2 → z2 - z4}

>> Barnes 1st Lemma will be checked for: {z5, z4, z3} <<
   Starting with dim=6 representation...


1. Checking z5
2. Checking z4...Barnes Lemma was applied.
3. Checking z3

>> Representation after 1st Barnes Lemma:  <<

   1st Barnes Lemma was applied for: {z4}
   Obtained representation has: dim=5
```

$$\Big\{\Big((-1)^{n1+n2+n3+n4+n5+n6}\,(m1^2)^{z1}\,(m2^2)^{z2}\,(m3^2)^{z6}\,(-s)^{4-2\,eps-n1-n2-n3-n4-n5-n6-z1-z2-z6}$$

```
     Gamma[-z1] Gamma[-z2] Gamma[2 - eps - n2 - n3 - z1 - z2 - z3] Gamma[-z3]
     Gamma[2 - eps - n2 - n6 - z1 - z2 - z5] Gamma[4 - 2 eps - 2 n2 - n3 - n6 - 2 z1 - z2 - z3 - z5]
     Gamma[-z5] Gamma[n2 + z1 + z3 + z5] Gamma[-2 + eps + n2 + n3 + n6 + z1 + z2 + z3 + z5]
     Gamma[2 - eps - n1 - n4 + z3 - z6] Gamma[2 - eps - n4 - n5 + z5 - z6]
     Gamma[-z6] Gamma[n4 + z6] Gamma[-2 + eps + n1 + n4 + n5 - z3 - z5 + z6]) /
   (Gamma[n2] Gamma[n3] Gamma[n4] Gamma[n6] Gamma[4 - 2 eps - n2 - n3 - n6 - z1 - z2]
     Gamma[n1 - z3] Gamma[n5 - z5] Gamma[4 - 2 eps - 2 n2 - n3 - n6 - 2 z1 - 2 z2 - z3 - z5]
     Gamma[4 - 2 eps - n1 - n4 - n5 + z3 + z5 - z6])}
```

```
finres = fin /. {n1 → 1, n2 → 1, n3 → 1, n4 → 1, n5 → 1, n6 → 1}
```

$$\Big\{\Big((m1^2)^{z1}\,(m2^2)^{z2}\,(m3^2)^{z6}\,(-s)^{-2-2\,eps-z1-z2-z6}\,\text{Gamma}[-z1]\,\text{Gamma}[-z2]\,\text{Gamma}[-eps-z1-z2-z3]\,\text{Gamma}[-z3]\,\text{Gamma}[-eps-z1-z2-z5]\,\text{Gamma}[-2\,eps-2\,z1-z2-z3-z5]\,\text{Gamma}[-z5]\,\text{Gamma}[1+z1+z3+z5]\,\text{Gamma}[1+eps+z1+z2+z3+z5]\,\text{Gamma}[-eps+z3-z6]\,\text{Gamma}[-eps+z5-z6]\,\text{Gamma}[-z6]\,\text{Gamma}[1+z6]\,\text{Gamma}[1+eps-z3-z5+z6]\Big)\Big/\left(\text{Gamma}[1-2\,eps-z1-z2]\,\text{Gamma}[1-z3]\,\text{Gamma}[1-z5]\,\text{Gamma}[-2\,eps-2\,z1-2\,z2-z3-z5]\,\text{Gamma}[1-2\,eps+z3+z5-z6]\right)\Big\}$$