Example#1, 2-loop planar vertex with three different masses in d=6-2*eps

The same diagram as in the AMBREv1.3_examples.nb file.
All computations are done in a fully automatic way.

```
<< AMBREv2.1.m
```

```
AMBRE by K.Kajda   ver: 2.1
last modified Jul 2016
```

```
<< PlanarityTestv1.2.m
```

```
by E. Dubovyk and K. Bielas ver: 1.2
created: January 2014
last executed: 25.07.2016 at 18:08
```

```
invariants = {p1^2 → 0, p2^2 → 0, p1 * p2 → s / 2};
```

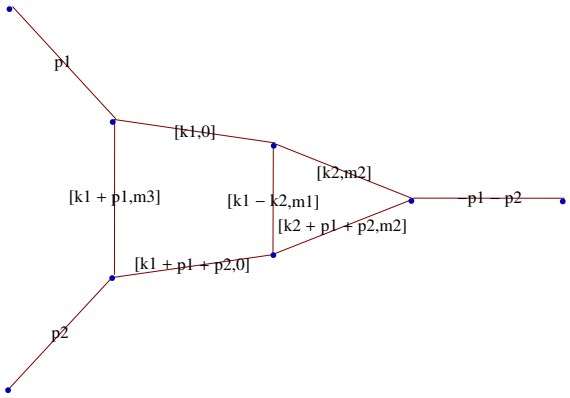
```
d = 6 - 2 eps;(* by default d=4-2 eps *)
```

```
ex = PR[k1, 0, n1] PR[k1 - k2, m1, n2] PR[k2, m2, n3]
    PR[k1 + p1, m3, n4] PR[k1 + p1 + p2, 0, n5] PR[k2 + p1 + p2, m2, n6];
```

```
PlanarityTest[{ex}, {k1, k2}, DrawGraph → True];
```

```
The Diagram
```

```
is planar.
```




```
repr = MBrepr[{1}, {ex}, {k2, k1}]
```

```
>>External momenta = N/A

>>Starting LoopByLoop calculation

--iteration nr: 1 with momentum: k2

  Run ?INT to see description of below output
{INT[{1}, 1, PR[k1 - k2, m1, n2] PR[k2, m2, n3] PR[k2 + p1 + p2, m2, n6], N/A]}

  F polynomial during this iteration
m1^2 X[1] + m2^2 X[2] - PR[k1, 0] X[1] X[2] + m2^2 X[3] - PR[k1 + p1 + p2, 0] X[1] X[3] - s X[2] X[3]

--iteration nr: 2 with momentum: k1

  Run ?INT to see description of below output

{INT[{1}, ((-1)^(1/2 (6-2 eps)+1/2 (-6+2 eps)+n2+n3+n6+z3+z5) (m1^2)^z1 (m2^2)^(z2+z4) (-s)^(1/2 (6-2 eps)-n2-n3-n6-z1-z2-z3-z4-z5)
      Gamma[-z1] Gamma[-z2] Gamma[3 - eps - n2 - n3 - z1 - z2 - z3]
      Gamma[-z3] Gamma[-z4] Gamma[3 - eps - n2 - n6 - z1 - z4 - z5] Gamma[-z5]
      Gamma[n2 + z1 + z3 + z5] Gamma[1/2 (-6 + 2 eps) + n2 + n3 + n6 + z1 + z2 + z3 + z4 + z5]) /
    (Gamma[n2] Gamma[n3] Gamma[n6] Gamma[6 - 2 eps - n2 - n3 - n6 - z1 - z2 - z4]),
   PR[k1, 0, n1 - z3] PR[k1 + p1, m3, n4] PR[k1 + p1 + p2, 0, n5 - z5],
   (-1)^(1/2 (6-2 eps)+1/2 (-6+2 eps)) s^(1/2 (6-2 eps)+1/2 (-6+2 eps))]}

  F polynomial during this iteration
m3^2 X[2] - s X[1] X[3]

>>Contracting and finalizing output

--contracting...

--finalizing output...

>>Checking Barnes 1-st lemma...

>> Shifting: {z2 → z2 - z4}

>> Barnes 1st Lemma will be checked for: {z5, z4, z3} <<
   Starting with dim=6 representation...


1. Checking z5
2. Checking z4...Barnes Lemma was applied.
3. Checking z3

>> Representation after 1st Barnes Lemma:  <<

   1st Barnes Lemma was applied for: {z4}
   Obtained representation has: dim=5
```

```
{{(-1)^(n1+n2+n3+n4+n5+n6) (m1^2)^z1 (m2^2)^z2 (m3^2)^z6 (-s)^(-2 eps-n1-n2-n3-n4-n5-n6-z1-z2-z6)
    s^6 Gamma[-z1] Gamma[-z2] Gamma[3 - eps - n2 - n3 - z1 - z2 - z3] Gamma[-z3]
    Gamma[3 - eps - n2 - n6 - z1 - z2 - z5] Gamma[6 - 2 eps - 2 n2 - n3 - n6 - 2 z1 - z2 - z3 - z5]
    Gamma[-z5] Gamma[n2 + z1 + z3 + z5] Gamma[-3 + eps + n2 + n3 + n6 + z1 + z2 + z3 + z5]
    Gamma[3 - eps - n1 - n4 + z3 - z6] Gamma[3 - eps - n4 - n5 + z5 - z6]
    Gamma[-z6] Gamma[n4 + z6] Gamma[-3 + eps + n1 + n4 + n5 - z3 - z5 + z6]) /
  (Gamma[n2] Gamma[n3] Gamma[n4] Gamma[n6] Gamma[6 - 2 eps - n2 - n3 - n6 - z1 - z2]
    Gamma[n1 - z3] Gamma[n5 - z5] Gamma[6 - 2 eps - 2 n2 - n3 - n6 - 2 z1 - 2 z2 - z3 - z5]
    Gamma[6 - 2 eps - n1 - n4 - n5 + z3 + z5 - z6])}
```

finres = repr /. {n1 → 1, n2 → 1, n3 → 1, n4 → 1, n5 → 1, n6 → 1}

```
{{(m1^2)^z1 (m2^2)^z2 (m3^2)^z6 (-s)^(-6-2 eps-z1-z2-z6) s^6
    Gamma[-z1] Gamma[-z2] Gamma[1 - eps - z1 - z2 - z3] Gamma[-z3]
    Gamma[1 - eps - z1 - z2 - z5] Gamma[2 - 2 eps - 2 z1 - z2 - z3 - z5] Gamma[-z5]
    Gamma[1 + z1 + z3 + z5] Gamma[eps + z1 + z2 + z3 + z5] Gamma[1 - eps + z3 - z6]
    Gamma[1 - eps + z5 - z6] Gamma[-z6] Gamma[1 + z6] Gamma[eps - z3 - z5 + z6]) /
  (Gamma[3 - 2 eps - z1 - z2] Gamma[1 - z3] Gamma[1 - z5]
    Gamma[2 - 2 eps - 2 z1 - 2 z2 - z3 - z5] Gamma[3 - 2 eps + z3 + z5 - z6])}
```

<< MB.m

```
MB 1.2

by Michal Czakon

improvements by Alexander Smirnov

more info in hep-ph/0511200

last modified 2 Jan 09
```

<< MBresolve.m

```
MBresolve 1.0

by Alexander Smirnov

more info in arXiv:0901.0386

last modified 4 Jan 09
```

step1 = MBresolve[#, eps] & /@ finres // Flatten;

```
CREATING RESIDUES LIST.........0.9601 seconds
EVALUATING RESIDUES.........0.0555 seconds
```

```
step2 = MBexpand[step1, Exp[2 * eps * EulerGamma], {eps, 0, 0}];
MBanalytic = MBmerge[step2]
```

$$\left\{\text{MBint}\left[\frac{1}{32\,\text{eps}^2}\left(4+18\,\text{eps}+\text{eps}^2\left(53+2\,\pi^2\right)-4\,\text{eps}\,(2+9\,\text{eps})\,\text{Log}\left[\text{m3}^2\right]+8\,\text{eps}^2\,\text{Log}\left[\text{m3}^2\right]^2\right),\right.\right.$$

$$\{\{\text{eps}\to 0\},\{\}\}\Big],\ \text{MBint}\Big[\frac{1}{16\,\text{eps}}\text{Gamma}[1-\text{z5}]\,\text{Gamma}[-\text{z5}]\,\text{Gamma}[\text{z5}]\,\text{Gamma}[1+\text{z5}]$$

$$\left(2+9\,\text{eps}-4\,\text{eps}\,\text{Log}\left[\text{m3}^2\right]-2\,\text{eps}\,\text{PolyGamma}[0,1-\text{z5}]+2\,\text{eps}\,\text{PolyGamma}[0,\text{z5}]\right),$$

$$\{\{\text{eps}\to 0\},\{\text{z5}\to 0.85806\}\}\Big],$$

$$\text{MBint}\Big[\frac{1}{2\,\text{eps}\,\text{Gamma}[3-\text{z6}]}\left(\text{m3}^2\right)^{\text{z6}}(-\text{s})^{-\text{z6}}\,\text{Gamma}[1-\text{z6}]^2\,\text{Gamma}[-\text{z6}]\,\text{Gamma}[\text{z6}]$$

$$\text{Gamma}[1+\text{z6}]\,(1+3\,\text{eps}+2\,\text{eps}\,\text{EulerGamma}-2\,\text{eps}\,\text{Log}[-\text{s}]-3\,\text{eps}\,\text{PolyGamma}[0,1-\text{z6}]+$$

$$3\,\text{eps}\,\text{PolyGamma}[0,3-\text{z6}]+2\,\text{eps}\,\text{PolyGamma}[0,\text{z6}]),$$

$$\{\{\text{eps}\to 0\},\{\text{z6}\to -0.224153\}\}\Big],\ \text{MBint}\Big[\Big(\left(\text{m3}^2\right)^{\text{z6}}(-\text{s})^{-\text{z6}}\,\text{Gamma}[-\text{z5}]\,\text{Gamma}[\text{z5}]$$

$$\text{Gamma}[1-\text{z5}-\text{z6}]\,\text{Gamma}[1+\text{z5}-\text{z6}]\,\text{Gamma}[-\text{z6}]\,\text{Gamma}[\text{z6}]\,\text{Gamma}[1+\text{z6}]\Big)\Big/$$

$$(2\,\text{Gamma}[3-\text{z6}]),\{\{\text{eps}\to 0\},\{\text{z5}\to 0.140245,\text{z6}\to -0.179148\}\}\Big],$$

$$\text{MBint}\Big[\Big(\left(\text{m2}^2\right)^{-\text{z3}-\text{z5}}\left(\text{m3}^2\right)^{\text{z6}}(-\text{s})^{\text{z3}+\text{z5}-\text{z6}}\,\text{Gamma}[-\text{z3}]\,\text{Gamma}[1+\text{z3}]\,\text{Gamma}[-\text{z5}]$$

$$\text{Gamma}[1+\text{z5}]\,\text{Gamma}[\text{z3}+\text{z5}]\,\text{Gamma}[1+\text{z3}+\text{z5}]\,\text{Gamma}[1+\text{z3}-\text{z6}]$$

$$\text{Gamma}[1+\text{z5}-\text{z6}]\,\text{Gamma}[-\text{z6}]\,\text{Gamma}[1+\text{z6}]\,\text{Gamma}[-\text{z3}-\text{z5}+\text{z6}]\Big)\Big/$$

$$(\text{Gamma}[1-\text{z3}]\,\text{Gamma}[1-\text{z5}]\,\text{Gamma}[2+\text{z3}+\text{z5}]\,\text{Gamma}[3+\text{z3}+\text{z5}]\,\text{Gamma}[3+\text{z3}+\text{z5}-\text{z6}]),$$

$$\{\{\text{eps}\to 0\},\{\text{z3}\to -0.276442,\text{z5}\to -0.459608,\text{z6}\to -0.446372\}\}\Big],\ \text{MBint}\Big[$$

$$\Big(\left(\text{m2}^2\right)^{\text{z2}}\left(\text{m3}^2\right)^{\text{z6}}(-\text{s})^{-\text{z2}-\text{z6}}\,\text{Gamma}[-\text{z2}]\,\text{Gamma}[1-\text{z2}-\text{z3}]\,\text{Gamma}[-\text{z3}]\,\text{Gamma}[1-\text{z2}-\text{z5}]$$

$$\text{Gamma}[2-\text{z2}-\text{z3}-\text{z5}]\,\text{Gamma}[-\text{z5}]\,\text{Gamma}[1+\text{z3}+\text{z5}]\,\text{Gamma}[\text{z2}+\text{z3}+\text{z5}]$$

$$\text{Gamma}[1+\text{z3}-\text{z6}]\,\text{Gamma}[1+\text{z5}-\text{z6}]\,\text{Gamma}[-\text{z6}]\,\text{Gamma}[1+\text{z6}]\,\text{Gamma}[-\text{z3}-\text{z5}+\text{z6}]\Big)\Big/$$

$$(\text{Gamma}[3-\text{z2}]\,\text{Gamma}[1-\text{z3}]\,\text{Gamma}[1-\text{z5}]\,\text{Gamma}[2-2\,\text{z2}-\text{z3}-\text{z5}]$$

$$\text{Gamma}[3+\text{z3}+\text{z5}-\text{z6}]),$$

$$\{\{\text{eps}\to 0\},\{\text{z2}\to -0.200233,\text{z3}\to -0.101036,\text{z5}\to -0.483143,\text{z6}\to -0.25871\}\}\Big],$$

$$\text{MBint}\Big[\Big(\left(\text{m1}^2\right)^{\text{z1}}\left(\text{m2}^2\right)^{\text{z2}}\left(\text{m3}^2\right)^{\text{z6}}(-\text{s})^{-\text{z1}-\text{z2}-\text{z6}}\,\text{Gamma}[-\text{z1}]\,\text{Gamma}[-\text{z2}]\,\text{Gamma}[1-\text{z1}-\text{z2}-\text{z3}]$$

$$\text{Gamma}[-\text{z3}]\,\text{Gamma}[1-\text{z1}-\text{z2}-\text{z5}]\,\text{Gamma}[2-2\,\text{z1}-\text{z2}-\text{z3}-\text{z5}]\,\text{Gamma}[-\text{z5}]$$

$$\text{Gamma}[1+\text{z1}+\text{z3}+\text{z5}]\,\text{Gamma}[\text{z1}+\text{z2}+\text{z3}+\text{z5}]\,\text{Gamma}[1+\text{z3}-\text{z6}]\,\text{Gamma}[1+\text{z5}-\text{z6}]$$

$$\text{Gamma}[-\text{z6}]\,\text{Gamma}[1+\text{z6}]\,\text{Gamma}[-\text{z3}-\text{z5}+\text{z6}]\Big)\Big/(\text{Gamma}[3-\text{z1}-\text{z2}]$$

$$\text{Gamma}[1-\text{z3}]\,\text{Gamma}[1-\text{z5}]\,\text{Gamma}[2-2\,\text{z1}-2\,\text{z2}-\text{z3}-\text{z5}]\,\text{Gamma}[3+\text{z3}+\text{z5}-\text{z6}]),$$

$$\{\{\text{eps}\to 0\},\{\text{z1}\to 0.507578,\text{z2}\to -0.272769,\text{z3}\to -0.0825234,$$

$$\left.\left.\text{z5}\to -0.0730761,\text{z6}\to -0.0666452\}\}\Big]\right\}$$

```
MBintegrate[MBanalytic, {s → -1, m1 → 1, m2 → 2, m3 → 3}]
```

```
Shifting contours...

Performing 4 lower-dimensional integrations with NIntegrate...1...2...3...4

Higher-dimensional integrals

Preparing MBpart1eps0 (dim 5)

Preparing MBpart2eps0 (dim 4)

Preparing MBpart3eps0 (dim 3)

Preparing MBpart4eps0 (dim 2)

Running MBpart1eps0

Running MBpart2eps0

Running MBpart3eps0

Running MBpart4eps0
```

$$\left\{0.216796 + \frac{0.125}{\text{eps}^2} - \frac{0.141395}{\text{eps}}, \{0.000274603, 0\}\right\}$$

```
Quit[]

<< ../FIESTA3.2/FIESTA3.m

UsingQLink = False; UsingC = False;

d0 = 6;
```

```
SDEvaluate[UF[{k1, k2}, {-k1^2, -(k1 - k2)^2 + m1^2, -k2^2 + m2^2, -(k1 + p1)^2 + m3^2,
   -(k1 + p1 + p2)^2, -(k2 + p1 + p2)^2 + m2^2}, {p1^2 → 0, p2^2 → 0, p1 * p2 → s / 2,
   s → -1, m1 → 1, m2 → 2, m3 → 3}], {1, 1, 1, 1, 1, 1}, 0]
```

```
FIESTA 3.2
Starting 1 subkernels
Subkernel will be used for launching external programs, all evaluations go on main kernel.
UsingC: False
NumberOfLinks: 1
UsingQLink: False
Strategy: STRATEGY_S
Integration has to be performed up to order 1
Sector decomposition - 6 sectors
Primary sector 1 resulted in 9 sectors.
Primary sector 2 resulted in 5 sectors.
Primary sector 3 resulted in 4 sectors.
Primary sector 4 resulted in 3 sectors.
Primary sector 5 resulted in 9 sectors.
Primary sector 6 resulted in 4 sectors.
Totally: 0.2702 seconds; 12 sectors.
Preparing database: 0.0009 seconds.
Variable substitution..........0.164 seconds; 34 terms.
Pole resolution..........0.0376 seconds; 49 terms.
Expression preparation..........0.0633 seconds; 49 terms.
Epsilon expansion..........0.0536 seconds; 113 terms.
Preparing integration strings..........0.117 seconds; 113 terms.
Database ready for integration.
Terms of order -1: 15, max vars: 4
Integrating..........5.9445 seconds.
Returned answer: 0.2500100709026885 + pm* 0.00002228915082049683
(0.12500503545134428 + 0.000011144575410248419*pm1)*ep^(-2)
Terms of order 0: 49, max vars: 5
Integrating..........16.1048 seconds.
Returned answer: -0.28287657366166613 + pm* 0
(0.12500503545134428 + 0.000011144575410248419*pm2)*ep^(-2)+(-0.1414382868308331)*ep^(-1)
Terms of order 1: 49, max vars: 5
Integrating..........20.4259 seconds.
Returned answer: -0.3893586055083057 + pm* 0
(0.12500503545134428 + 0.000011144575410248419*pm4)*ep^(-2)+(-0.1414382868308331)*ep^(-1)+
Total integration time: 42.4838
Total time used: 43.3877 seconds.
```

$$0.216571 - \frac{0.141438}{\text{ep}} + \frac{0.125005 + 0.0000111446\,\text{pm7}}{\text{ep}^2} + 0.0000366642\,\text{pm9}$$

```
Quit[]
```