Example#1, calculation in d=6-2*eps

```
<< AMBREv3.1.m
```

```
AMBRE v3.1 [Jul 2016] by I.Dubovyk, http://us.edu.pl/~gluza/ambre/
License: http://creativecommons.org, CC BY-ND
Ref.: I. Dubovyk, J. Gluza, T. Riemann, J. Usovitsch, arXiv:1607.07538.
```

```
<< PlanarityTestv1.2.m
```

```
by E. Dubovyk and K. Bielas ver: 1.2
created: January 2014
last executed: 22.07.2016 at 17:37
```

```
invariants = {p1^2 → 0, p2^2 → 0, p1 * p2 → s / 2};
```

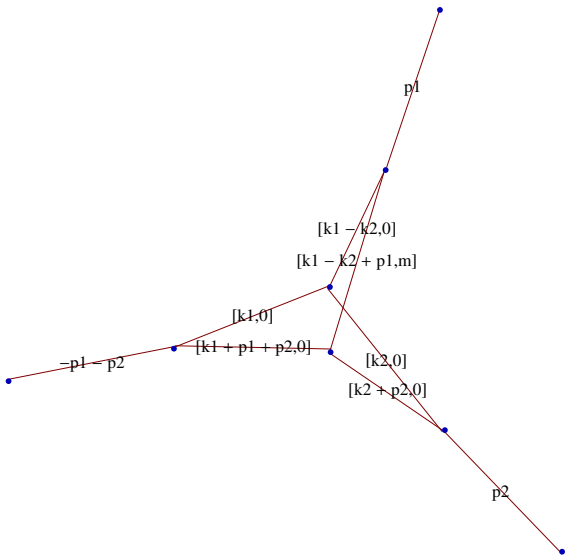
```
d = 6 - 2 eps;(* by default d=4-2 eps *)
```

```
ex1 = PR[k1, 0, n1] PR[k1 - k2, 0, n2] PR[k2, 0, n3]
    PR[k1 - k2 + p1, m, n4] PR[k2 + p2, 0, n5] PR[k1 + p1 + p2, 0, n6];
```

```
PlanarityTest[{ex1}, {k1, k2}, DrawGraph → True];
```

```
The Diagram
```

```
is non-planar.
```




```
res = MBreprNP[{1}, {ex1}, {k1, k2}]
```

```
Fauto::mode : F polynomial will be calculated in AUTO mode. In order to use MANUAL mode execute Fauto[0].
```

```
Upoly = x[1] x[2] + x[1] x[3] + x[2] x[3] + x[1] x[4] + x[3] x[4] +
  x[1] x[5] + x[2] x[5] + x[4] x[5] + x[2] x[6] + x[3] x[6] + x[4] x[6] + x[5] x[6]

Fpoly = m^2 Upoly x[4] - s x[1] x[4] x[5] - s x[1] x[2] x[6] -
  s x[1] x[3] x[6] - s x[2] x[3] x[6] - s x[1] x[4] x[6] - s x[1] x[5] x[6]

{((-1)^(n1+n2+n3+n4+n5+n6) (m^2)^z1 (-s)^(6-2 eps-n1-n2-n3-n4-n5-n6-z1) Gamma[3 - eps - n3 - n5]
    Gamma[3 - eps - n2 - n4 - z1] Gamma[-z1] Gamma[3 - eps - n1 - n6 - z2]
    Gamma[-z2] Gamma[6 - 2 eps - n2 - n3 - n4 - n5 - n6 - z1 - z3]
    Gamma[6 - 2 eps - n1 - n2 - n3 - n5 - n6 - z2 - z3]
    Gamma[6 - 2 eps - n1 - n2 - n3 - n4 - n6 - z1 - z2 - z3]
    Gamma[-z3] Gamma[n2 + z3] Gamma[n3 + z3] Gamma[n6 + z2 + z3]
    Gamma[-6 + 2 eps + n1 + n2 + n3 + n4 + n5 + n6 + z1 + z2 + z3]) /
  (Gamma[n1] Gamma[n2] Gamma[n3] Gamma[n4] Gamma[n5] Gamma[n6]
    Gamma[6 - 2 eps - n2 - n3 - n4 - n5 - z1] Gamma[9 - 3 eps - n1 - n2 - n3 - n4 - n5 - n6 - z1]
    Gamma[6 - 2 eps - n1 - n3 - n5 - n6 - z2] Gamma[6 - 2 eps - n1 - n2 - n4 - n6 - z1 - z2])}
```

finres = res /. {n1 → 1, n2 → 1, n3 → 1, n4 → 1, n5 → 1, n6 → 1}

```
{((m^2)^z1 (-s)^(-2 eps-z1) Gamma[1 - eps] Gamma[1 - eps - z1]
    Gamma[-z1] Gamma[1 - eps - z2] Gamma[-z2] Gamma[1 - 2 eps - z1 - z3]
    Gamma[1 - 2 eps - z2 - z3] Gamma[1 - 2 eps - z1 - z2 - z3] Gamma[-z3]
    Gamma[1 + z3]^2 Gamma[1 + z2 + z3] Gamma[2 eps + z1 + z2 + z3]) /
  (Gamma[3 - 3 eps - z1] Gamma[2 - 2 eps - z1] Gamma[2 - 2 eps - z2] Gamma[2 - 2 eps - z1 - z2])}
```

<< MB.m

```
MB 1.2

by Michal Czakon

improvements by Alexander Smirnov

more info in hep-ph/0511200

last modified 2 Jan 09
```

<< MBresolve.m

```
MBresolve 1.0

by Alexander Smirnov

more info in arXiv:0901.0386

last modified 4 Jan 09
```

step1 = MBresolve[#, eps] & /@ finres // Flatten;

```
CREATING RESIDUES LIST.....1.3825 seconds
EVALUATING RESIDUES.....0.0139 seconds
```

```
step2 = MBexpand[step1, Exp[2 * eps * EulerGamma], {eps, 0, 0}];
MBanalytic = MBmerge[step2]
```

$$\left\{\text{MBint}\left[\frac{1}{8}\left(21+\frac{2}{\text{eps}}-4\,\text{Log}[-s]\right),\{\{\text{eps}\to 0\},\{\}\}\right],\right.$$
$$\text{MBint}\left[\left(\left(m^2\right)^{-z3}(-s)^{z3}\,\text{Gamma}[1-z3]\,\text{Gamma}[-z3]\,\text{Gamma}[z3]\,\text{Gamma}[1+z3]^4\right)\Big/\right.$$
$$\left.\left(\text{Gamma}[2+z3]^2\,\text{Gamma}[3+z3]\right),\{\{\text{eps}\to 0\},\{z3\to -0.224153\}\}\right],$$
$$\text{MBint}\left[\left(\left(m^2\right)^{-z2-z3}(-s)^{z2+z3}\,\text{Gamma}[1-z2]\,\text{Gamma}[-z2]\,\text{Gamma}[1+z2]\,\text{Gamma}[1-z2-z3]\right.\right.$$
$$\left.\text{Gamma}[-z3]\,\text{Gamma}[1+z3]^2\,\text{Gamma}[z2+z3]\,\text{Gamma}[1+z2+z3]^2\right)\Big/$$
$$\left.(\text{Gamma}[2-z2]\,\text{Gamma}[2+z3]\,\text{Gamma}[2+z2+z3]\,\text{Gamma}[3+z2+z3]),\right.$$
$$\left.\{\{\text{eps}\to 0\},\{z2\to 0.747011,\,z3\to -0.115593\}\}\right],$$
$$\text{MBint}\left[\left(\left(m^2\right)^{z1}(-s)^{-z1}\,\text{Gamma}[1-z1]\,\text{Gamma}[-z1]\,\text{Gamma}[1-z2]\,\text{Gamma}[-z2]\,\text{Gamma}[1-z1-z3]\right.\right.$$
$$\left.\text{Gamma}[1-z2-z3]\,\text{Gamma}[1-z1-z2-z3]\,\text{Gamma}[-z3]\,\text{Gamma}[1+z3]^2\,\text{Gamma}[1+z2+z3]\right.$$
$$\left.\left.\text{Gamma}[z1+z2+z3]\right)\Big/(\text{Gamma}[2-z1]\,\text{Gamma}[3-z1]\,\text{Gamma}[2-z2]\,\text{Gamma}[2-z1-z2]),\right.$$
$$\left.\left.\{\{\text{eps}\to 0\},\{z1\to -0.166718,\,z2\to -0.433623,\,z3\to -0.118288\}\}\right]\right\}$$

```
MBintegrate[MBanalytic, {s → -1, m → 1}]
```

```
Shifting contours...

Performing 1 lower-dimensional integrations with NIntegrate...1

Higher-dimensional integrals

Preparing MBpart1eps0 (dim 3)

Preparing MBpart2eps0 (dim 2)

Running MBpart1eps0

Running MBpart2eps0
```

$$\left\{0.504571+\frac{0.25}{\text{eps}},\{0.0000242912,\,0\}\right\}$$

```
Quit[]

<< ../../FIESTA3.2/FIESTA3.m

UsingQLink = False; UsingC = False;

d0 = 6;

SDEvaluate[UF[{k1, k2}, {-k1^2, -(k1 - k2)^2, -k2^2, -(k1 - k2 + p1)^2 + m^2,
    -(k2 + p2)^2, -(k1 + p1 + p2)^2}, {p1^2 → 0, p2^2 → 0, p1 * p2 → s / 2,
    s → -1, m → 1}], {1, 1, 1, 1, 1, 1}, 0]

Quit[]
```

Example#2, 3-loop non-planar box (arXiv:1312.2588)

```
<< AMBREv3.1.m
```

```
AMBRE v3.1 [Jul 2016] by I.Dubovyk, http://us.edu.pl/~gluza/ambre/
License: http://creativecommons.org, CC BY-ND
Ref.: I. Dubovyk, J. Gluza, T. Riemann, J. Usovitsch, arXiv:1607.07538.
```

```
<< PlanarityTestv1.2.m
```

```
by E. Dubovyk and K. Bielas ver: 1.2
created: January 2014
last executed: 25.07.2016 at 16:01
```

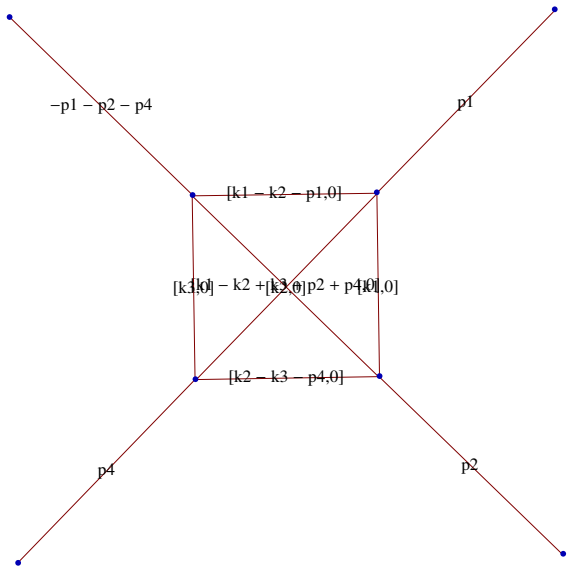
```
invariants = {p1^2 → 0, p2^2 → 0, p3^2 → 0, p4^2 → 0, p1 * p2 → 1 / 2 * s, p3 * p4 → 1 / 2 s,
    p1 * p3 → 1 / 2 * t, p2 * p4 → 1 / 2 * t, p2 * p3 → 1 / 2 * u, p1 * p4 → 1 / 2 * u};
```

```
ex2 = PR[k1, 0, n1] PR[k2, 0, n2] PR[k3, 0, n3]
    PR[k1 - k2 - p1, 0, n4] PR[k2 - k3 - p4, 0, n5] PR[k1 - k2 + k3 + p2 + p4, 0, n6];
```

```
PlanarityTest[{ex2}, {k1, k2, k3}, DrawGraph → True];
```

```
The Diagram
```

```
is non-planar.
```




```
SimplifyFpoly[{s + t + u → 0}];
```

```
res = MBreprNP[{1}, {ex2}, {k1, k2, k3}]
```

Fauto::mode : F polynomial will be calculated in AUTO mode. In order to use MANUAL mode execute Fauto[0].

```
Upoly = x[1] x[2] x[3] + x[1] x[3] x[4] + x[2] x[3] x[4] + x[1] x[2] x[5] + x[1] x[3] x[5] +
  x[1] x[4] x[5] + x[2] x[4] x[5] + x[3] x[4] x[5] + x[1] x[2] x[6] + x[1] x[3] x[6] +
  x[2] x[3] x[6] + x[1] x[4] x[6] + x[2] x[4] x[6] + x[2] x[5] x[6] + x[3] x[5] x[6] + x[4] x[5] x[6]

Fpoly = -u x[1] x[3] x[4] x[5] - t x[1] x[2] x[3] x[6] - s x[2] x[4] x[5] x[6]
```

$$\left\{\left((-1)^{n1+n2+n3+n4+n5+n6}\ (-s)^{z1}\ (-t)^{\frac{3}{2}(4-2\,eps)-n1-n2-n3-n4-n5-n6-z1-z2}\right.\right.$$

$$(-u)^{z2}\ \text{Gamma}\left[-\frac{3}{2}\ (4-2\,eps)+n1+n2+n3+n4+n5+n6\right]\ \text{Gamma}[-z1]$$

$$\text{Gamma}[-z2]\ \text{Gamma}[-6+3\,eps+n1+n2+n3+n4+n5+n6+z1+z2]\ \text{Gamma}[-z4]$$

$$\text{Gamma}[6-3\,eps-n1-n2-n4-n5-n6-z1+z5]\ \text{Gamma}[2-eps-n2+z2+z3+z5]$$

$$\text{Gamma}[6-3\,eps-n2-n3-n4-n5-n6-z1+z3+z4+z5]$$

$$\text{Gamma}[-4+2\,eps+n1+n4+n5+n6+z1+z2+z3-z6]$$

$$\text{Gamma}[-8+4\,eps+n1+n2+n3+n4+2\,n5+n6+z1+z2-z3-z4-z5-z6]$$

$$\text{Gamma}[-z6]\ \text{Gamma}[6-3\,eps-n1-n3-n4-n5-n6-z2+z4+z6]$$

$$\text{Gamma}[8-4\,eps-n1-n2-n3-n4-2\,n5-n6-z1-z2+z4+z5+z6]$$

$$\text{Gamma}[2-eps-n1-n4-z2-z3-z7]\ \text{Gamma}[-z7]$$

$$\left.\text{Gamma}[-z5+z7]\ \text{Gamma}[4-2\,eps-n1-n5-n6+z6+z7]\right)\Big/$$

$$(\text{Gamma}[n1]\ \text{Gamma}[n2]\ \text{Gamma}[n3]\ \text{Gamma}[n4]\ \text{Gamma}[n5]$$

$$\text{Gamma}[8-4\,eps-n1-n2-n3-n4-n5-n6]\ \text{Gamma}[n6]$$

$$\text{Gamma}[-6+3\,eps+n1+n2+n3+n4+n5+n6]\ \text{Gamma}[2-eps-n2+z2+z3+z5-z6]$$

$$\left.\left.\text{Gamma}[12-6\,eps-2\,n1-n2-n3-2\,n4-2\,n5-2\,n6-z1-z2+z4+z5+z6])\right\}\right.$$

```
fin = BarnesLemma[#, 1] & /@ res
```

```
>> Barnes 1st Lemma will be checked for: {z7, z6, z5, z4, z3} <<
   Starting with dim=7 representation...


1. Checking z7...Barnes Lemma was applied.
2. Checking z6
3. Checking z5
4. Checking z4
5. Checking z3
>> Representation after 1st Barnes Lemma:  <<
   1st Barnes Lemma was applied for: {z7}
   Obtained representation has: dim=6
```

$$\left\{\left((-1)^{n1+n2+n3+n4+n5+n6}\ (-s)^{z1}\ (-t)^{\frac{3}{2}(4-2\,eps)-n1-n2-n3-n4-n5-n6-z1-z2}\ (-u)^{z2}\right.\right.$$
$$\Gamma[-z1]\,\Gamma[-z2]\,\Gamma[-6+3\,eps+n1+n2+n3+n4+n5+n6+z1+z2]$$
$$\Gamma[-z4]\,\Gamma[2-eps-n1-n4-z2-z3-z5]\,\Gamma[-z5]$$
$$\Gamma[6-3\,eps-n1-n2-n4-n5-n6-z1+z5]\,\Gamma[2-eps-n2+z2+z3+z5]$$
$$\Gamma[6-3\,eps-n2-n3-n4-n5-n6-z1+z3+z4+z5]$$
$$\Gamma[-4+2\,eps+n1+n4+n5+n6+z1+z2+z3-z6]$$
$$\Gamma[-8+4\,eps+n1+n2+n3+n4+2\,n5+n6+z1+z2-z3-z4-z5-z6]\,\Gamma[-z6]$$
$$\Gamma[4-2\,eps-n1-n5-n6+z6]\,\Gamma[6-3\,eps-2\,n1-n4-n5-n6-z2-z3+z6]$$
$$\Gamma[6-3\,eps-n1-n3-n4-n5-n6-z2+z4+z6]$$
$$\left.\Gamma[8-4\,eps-n1-n2-n3-n4-2\,n5-n6-z1-z2+z4+z5+z6]\right)\Big/$$
$$(\Gamma[n1]\,\Gamma[n2]\,\Gamma[n3]\,\Gamma[n4]\,\Gamma[n5]$$
$$\Gamma[8-4\,eps-n1-n2-n3-n4-n5-n6]\,\Gamma[n6]$$
$$\Gamma[2-eps-n2+z2+z3+z5-z6]\,\Gamma[6-3\,eps-2\,n1-n4-n5-n6-z2-z3-z5+z6]$$
$$\left.\left.\Gamma[12-6\,eps-2\,n1-n2-n3-2\,n4-2\,n5-2\,n6-z1-z2+z4+z5+z6]\right)\right\}$$

finres = fin /. {n1 → 1, n2 → 1, n3 → 1, n4 → 1, n5 → 1, n6 → 1}

$$\left\{\left((-s)^{z1}\ (-t)^{-6+\frac{3}{2}(4-2\,eps)-z1-z2}\ (-u)^{z2}\,\Gamma[-z1]\,\Gamma[-z2]\,\Gamma[3\,eps+z1+z2]\right.\right.$$
$$\Gamma[-z4]\,\Gamma[-eps-z2-z3-z5]\,\Gamma[-z5]\,\Gamma[1-3\,eps-z1+z5]$$
$$\Gamma[1-eps+z2+z3+z5]\,\Gamma[1-3\,eps-z1+z3+z4+z5]$$
$$\Gamma[2\,eps+z1+z2+z3-z6]\,\Gamma[-1+4\,eps+z1+z2-z3-z4-z5-z6]$$
$$\Gamma[-z6]\,\Gamma[1-2\,eps+z6]\,\Gamma[1-3\,eps-z2-z3+z6]$$
$$\left.\Gamma[1-3\,eps-z2+z4+z6]\,\Gamma[1-4\,eps-z1-z2+z4+z5+z6]\right)\Big/$$
$$(\Gamma[2-4\,eps]\,\Gamma[1-eps+z2+z3+z5-z6]\,\Gamma[1-3\,eps-z2-z3-z5+z6]$$
$$\left.\left.\Gamma[2-6\,eps-z1-z2+z4+z5+z6]\right)\right\}$$

```
<< MB.m
MB 1.2
by Michal Czakon
improvements by Alexander Smirnov
more info in hep-ph/0511200
last modified 2 Jan 09
<< MBresolve.m
```

```
MBresolve 1.0

by Alexander Smirnov

more info in arXiv:0901.0386

last modified 4 Jan 09
```

```
step1 = MBresolve[#, eps] & /@ finres // Flatten;
```

```
CREATING RESIDUES LIST..........0.889 seconds
EVALUATING RESIDUES..........0.0239 seconds
```

```
step2 = MBexpand[step1, Exp[3 * eps * EulerGamma], {eps, 0, 0}];
MBanalytic = MBmerge[step2]
```

```
{MBint[-(Gamma[-z4] Gamma[-z3 - z5] Gamma[-z5] Gamma[1 + z5] Gamma[1 + z3 + z5]
       Gamma[1 + z3 + z4 + z5] Gamma[z3 - z6] Gamma[-1 - z3 - z4 - z5 - z6] Gamma[-z6]
       Gamma[1 + z6] Gamma[1 - z3 + z6] Gamma[1 + z4 + z6] Gamma[1 + z4 + z5 + z6]
       (-1 - 4 eps + 4 eps EulerGamma + 3 eps Log[-s] + eps PolyGamma[0, -z3 - z5] +
          eps PolyGamma[0, 1 + z3 + z5] + eps PolyGamma[0, z3 - z6] -
          eps PolyGamma[0, -1 - z3 - z4 - z5 - z6] - eps PolyGamma[0, 1 + z3 + z5 - z6] +
          2 eps PolyGamma[0, 1 + z6] + 3 eps PolyGamma[0, 1 - z3 + z6] +
          3 eps PolyGamma[0, 1 + z4 + z6] - 3 eps PolyGamma[0, 1 - z3 - z5 + z6] +
          eps PolyGamma[0, 1 + z4 + z5 + z6] - 3 eps PolyGamma[0, 2 + z4 + z5 + z6])) /
    (3 eps Gamma[1 + z3 + z5 - z6] Gamma[1 - z3 - z5 + z6] Gamma[2 + z4 + z5 + z6]),
  {{eps → 0}, {z3 → -0.190272, z4 → -0.29106,
    z5 → -0.125162, z6 → -0.412089}}],
 MBint[((-s)^z1 (-t)^-z1 Gamma[-z1] Gamma[z1] Gamma[-z4] Gamma[-z3 - z5]
      Gamma[-z5] Gamma[1 - z1 + z5] Gamma[1 + z3 + z5] Gamma[1 - z1 + z3 + z4 + z5]
      Gamma[z1 + z3 - z6] Gamma[-1 + z1 - z3 - z4 - z5 - z6] Gamma[-z6] Gamma[1 + z6]
      Gamma[1 - z3 + z6] Gamma[1 + z4 + z6] Gamma[1 - z1 + z4 + z5 + z6]) /
    (Gamma[1 + z3 + z5 - z6] Gamma[1 - z3 - z5 + z6] Gamma[2 - z1 + z4 + z5 + z6]),
  {{eps → 0}, {z1 → -0.28024, z3 → -0.168026, z4 → -0.0586432,
    z5 → -0.446559, z6 → -0.74032}}],
 MBint[((-s)^z1 (-t)^-z1-z2 (-u)^z2 Gamma[-z1] Gamma[-z2] Gamma[z1 + z2]
      Gamma[-z4] Gamma[-z2 - z3 - z5] Gamma[-z5] Gamma[1 - z1 + z5]
      Gamma[1 + z2 + z3 + z5] Gamma[1 - z1 + z3 + z4 + z5] Gamma[z1 + z2 + z3 - z6]
      Gamma[-1 + z1 + z2 - z3 - z4 - z5 - z6] Gamma[-z6] Gamma[1 + z6]
      Gamma[1 - z2 - z3 + z6] Gamma[1 - z2 + z4 + z6] Gamma[1 - z1 - z2 + z4 + z5 + z6]) /
    (Gamma[1 + z2 + z3 + z5 - z6] Gamma[1 - z2 - z3 - z5 + z6] Gamma[2 - z1 - z2 + z4 + z5 + z6]),
  {{eps → 0}, {z1 → -0.274696, z2 → 0.457648, z3 → -0.33939,
    z4 → -0.175791, z5 → -0.198596, z6 → -0.355397}}]}
```

```
MBintegrate[MBanalytic, {s → -1, t → -1, u → -1}]
```

```
Shifting contours...

Performing 0 lower-dimensional integrations with NIntegrate

Higher-dimensional integrals

Preparing MBpart1eps0 (dim 6)

Preparing MBpart2eps0 (dim 5)

Preparing MBpart3eps0 (dim 4)

Preparing MBpart4eps-1 (dim 4)

Running MBpart1eps0

Running MBpart2eps0

Running MBpart3eps0

Running MBpart4eps-1
```

$$\left\{26.6384 + \frac{2.40343}{\text{eps}}, \left\{0.0591813 + \frac{0.00203386}{\text{eps}}, 0\right\}\right\}$$

```
Quit[]

<< ../../FIESTA3.2/FIESTA3.m

UsingQLink = False; UsingC = False;

UFpoly = UF[{k1, k2, k3}, {-k1^2, -k2^2, -k3^2,
      -(k1 - k2 - p1)^2, -(k2 - k3 - p4)^2, -(k1 - k2 + k3 + p2 + p4)^2},
     {p1^2 → 0, p2^2 → 0, p3^2 → 0, p4^2 → 0, p1 * p2 → 1 / 2 * s, p3 * p4 → 1 / 2 s,
      p1 * p3 → 1 / 2 * t, p2 * p4 → 1 / 2 * t, p2 * p3 → 1 / 2 * u, p1 * p4 → 1 / 2 * u}] //.
   {s x_. + t x_. + u x_. → 0} /. {s → -1, t → -1, u → -1}

{x[1] x[2] x[3] + x[1] x[3] x[4] + x[2] x[3] x[4] + x[1] x[2] x[5] +
  x[1] x[3] x[5] + x[1] x[4] x[5] + x[2] x[4] x[5] + x[3] x[4] x[5] +
  x[1] x[2] x[6] + x[1] x[3] x[6] + x[2] x[3] x[6] + x[1] x[4] x[6] +
  x[2] x[4] x[6] + x[2] x[5] x[6] + x[3] x[5] x[6] + x[4] x[5] x[6],
 x[1] x[3] x[4] x[5] + x[1] x[2] x[3] x[6] + x[2] x[4] x[5] x[6], 3}
```

```
SDEvaluate[UFpoly, {1, 1, 1, 1, 1, 1}, 0]
```

```
FIESTA 3.2
Starting 1 subkernels
Subkernel will be used for launching external programs, all evaluations go on main kernel.
UsingC: False
NumberOfLinks: 1
UsingQLink: False
Strategy: STRATEGY_S
Integration has to be performed up to order 1
Sector decomposition - 6 sectors
Primary sector 1 resulted in 72 sectors.
Primary sector 2 resulted in 72 sectors.
Primary sector 3 resulted in 72 sectors.
Primary sector 4 resulted in 72 sectors.
Primary sector 5 resulted in 72 sectors.
Primary sector 6 resulted in 72 sectors.
Totally: 1.5544 seconds; 12 sectors.
Preparing database: 0.009 seconds.
Variable substitution..........0.8576 seconds; 432 terms.
Pole resolution..........0.2587 seconds; 432 terms.
Expression preparation..........0.2424 seconds; 432 terms.
Epsilon expansion..........0.3853 seconds; 864 terms.
Preparing integration strings..........2.0875 seconds; 864 terms.
Database ready for integration.
Terms of order 0: 432, max vars: 5
Integrating..........195.1828 seconds.
Returned answer: 7.211648565561291 + pm* 0.000050859911085447316
(2.403882855187097 + 0.00001695330369514910 5*pm1)*ep^(-1)
Terms of order 1: 432, max vars: 5
Integrating..........224.1377 seconds.
Returned answer: 79.59012462632185 + pm* 0
(2.403882855187097 + 0.00001695330369514910 5*pm2)*ep^(-1)+(26.530041542107277)*1
Total integration time: 419.3269
Total time used: 424.821 seconds.
```

$$26.53 + \frac{2.40388 + 0.0000169533\,\text{pm4}}{\text{ep}}$$

```
Quit[]
```

Example#3, 3-loop non-planar massless vertex

To run this example, please modify the MBresolve.m file:
in the line 60 must be
point=NMinimize[function,vars,Method -> “DifferentialEvolution”, **MaxIterations->1000**];
(be aware, it may take several hours to get the result)

```
<< AMBREv3.1.m
```

```
AMBRE v3.1 [Jul 2016] by I.Dubovyk, http://us.edu.pl/~gluza/ambre/
License: http://creativecommons.org, CC BY-ND
Ref.: I. Dubovyk, J. Gluza, T. Riemann, J. Usovitsch, arXiv:1607.07538.
```

```
<< PlanarityTestv1.2.m
```

```
by E. Dubovyk and K. Bielas ver: 1.2
created: January 2014
last executed: 27.07.2016 at 23:10
```

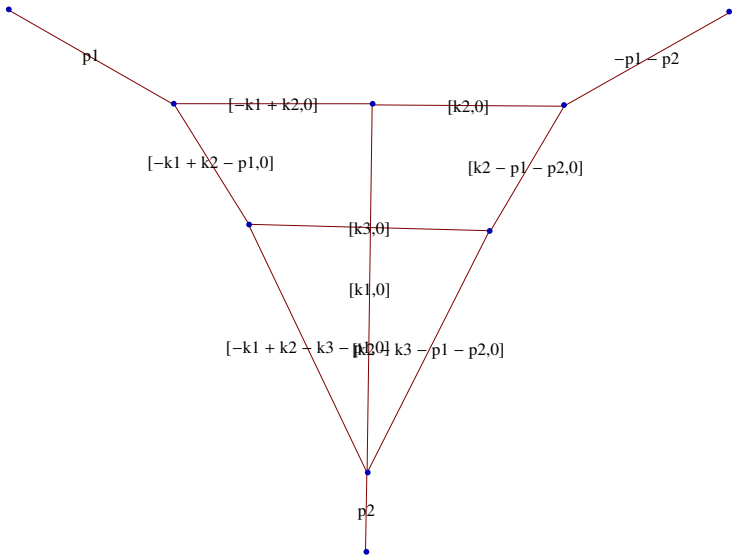
```
invariants = {p1^2 → 0, p2^2 → 0, p1 * p2 → s / 2};

ex4 = PR[k1, 0, n1] PR[k2, 0, n2] PR[k3, 0, n3] PR[k2 - k1, 0, n4] PR[k2 - k1 - p1, 0, n5]
    PR[k2 - p1 - p2, 0, n6] PR[k2 - k3 - p1 - p2, 0, n7] PR[k2 - k1 - k3 - p1, 0, n8];

PlanarityTest[{ex4}, {k1, k2, k3}, DrawGraph → True];
```

The Diagram

is non-planar.




```
res = MBreprNP[{1}, {ex4}, {k1, k2, k3}]
```

Fauto::mode : F polynomial will be calculated in AUTO mode. In order to use MANUAL mode execute Fauto[0].

```
Upoly = x[1] x[2] x[4] + x[1] x[3] x[4] + x[2] x[3] x[4] + x[1] x[4] x[5] + x[2] x[4] x[5] +
  x[1] x[2] x[6] + x[1] x[3] x[6] + x[2] x[3] x[6] + x[1] x[4] x[6] + x[2] x[4] x[6] +
  x[1] x[5] x[6] + x[2] x[5] x[6] + x[1] x[4] x[7] + x[3] x[4] x[7] + x[4] x[5] x[7] +
  x[1] x[6] x[7] + x[3] x[6] x[7] + x[4] x[6] x[7] + x[5] x[6] x[7] + x[1] x[2] x[8] +
  x[1] x[3] x[8] + x[2] x[3] x[8] + x[1] x[4] x[8] + x[3] x[4] x[8] + x[1] x[5] x[8] +
  x[2] x[5] x[8] + x[4] x[5] x[8] + x[2] x[6] x[8] + x[3] x[6] x[8] + x[4] x[6] x[8] +
  x[5] x[6] x[8] + x[1] x[7] x[8] + x[3] x[7] x[8] + x[5] x[7] x[8] + x[6] x[7] x[8]

Fpoly =
 - s x[1] x[2] x[4] x[7] - s x[1] x[3] x[4] x[7] - s x[2] x[3] x[4] x[7] - s x[2] x[4] x[5] x[7] -
  s x[1] x[2] x[6] x[7] - s x[1] x[3] x[6] x[7] - s x[2] x[3] x[6] x[7] - s x[2] x[4] x[6] x[7] -
  s x[2] x[5] x[6] x[7] - s x[1] x[2] x[4] x[8] - s x[1] x[3] x[4] x[8] - s x[2] x[3] x[4] x[8] -
  s x[2] x[4] x[5] x[8] - s x[2] x[4] x[6] x[8] - s x[2] x[5] x[6] x[8] - s x[1] x[2] x[7] x[8] -
  s x[1] x[3] x[7] x[8] - s x[2] x[3] x[7] x[8] - s x[2] x[5] x[7] x[8] - s x[2] x[6] x[7] x[8]

{((-1)^(n1+n2+n3+n4+n5+n6+n7+n8) (-s)^(3/2 (4-2 eps)-n1-n2-n3-n4-n5-n6-n7-n8

    Gamma[-3/2 (4 - 2 eps) + n1 + n2 + n3 + n4 + n5 + n6 + n7 + n8] Gamma[-z1]
    Gamma[-z11] Gamma[-z13] Gamma[-z14] Gamma[-z12 + z14] Gamma[-z2]
    Gamma[6 - 3 eps - n1 - n2 - n3 - n4 - n5 - n7 - n8 - z1 - z2 - z3] Gamma[-z3] Gamma[n5 + z3]
    Gamma[-z4] Gamma[-z5] Gamma[6 - 3 eps - n1 - n3 - n4 - n5 - n6 - n7 - n8 - z1 - z4 - z5 - z6]
    Gamma[2 - eps - n1 - n4 - n5 - z1 - z10 - z14 - z4 - z5 - z6]
    Gamma[12 - 6 eps - 2 n1 - n2 - 2 n3 - 2 n4 - 2 n5 - n6 - 2 n7 - 2 n8 - 2 z1 + z11 +
      z13 - z2 - z3 - z4 - z5 - z6] Gamma[-z6] Gamma[n4 + z1 + z4 + z5 + z6]
    Gamma[-4 + 2 eps + n1 + n3 + n4 + n5 + n7 + n8 + 2 z1 + z10 + z12 + z2 + z3 + z4 + z5 + z6] Gamma[
     8 - 4 eps - n1 - n2 - n3 - n4 - n5 - n6 - 2 n7 - n8 - z1 + z11 + z12 + z13 - z2 - z3 - z4 - z7]
    Gamma[-z7] Gamma[-8 + 4 eps + n1 + n2 + n3 + n4 + n5 + n6 + 2 n7 + n8 + z1 - z10 -
      z11 - z12 - z13 + z2 + z3 + z4 + z7] Gamma[-z8] Gamma[10 - 5 eps - 2 n1 - n2 -
      n3 - n4 - n5 - n6 - 2 n7 - 2 n8 - z1 + z13 + z14 - z2 - z3 - z4 - z5 - z7 - z9]
    Gamma[6 - 3 eps - n1 - n2 - n4 - n5 - n6 - n7 - n8 + z12 - z3 - z4 - z5 - z7 - z8 - z9]
    Gamma[-z9] Gamma[n1 + z1 + z10 + z11 + z12 + z4 + z5 + z6 + z9] Gamma[
     -6 + 3 eps + n1 + n2 + n3 + n4 + n5 + n6 + n7 + n8 + z1 + z2 + z3 + z4 + z5 + z6 + z7 + z8 + z9]
    Gamma[-10 + 5 eps + 2 n1 + n2 + n3 + 2 n4 + 2 n5 + n6 + 2 n7 + 2 n8 +
      2 z1 + z10 - z13 + z2 + 2 z3 + 2 z4 + 2 z5 + z6 + z7 + z8 + z9]) /

  (Gamma[n1] Gamma[n2] Gamma[n3] Gamma[n4] Gamma[n5] Gamma[n6] Gamma[n7]
    Gamma[8 - 4 eps - n1 - n2 - n3 - n4 - n5 - n6 - n7 - n8] Gamma[n8]
    Gamma[-6 + 3 eps + n1 + n2 + n3 + n4 + n5 + n6 + n7 + n8]
    Gamma[12 - 6 eps - 2 n1 - n2 - 2 n3 - 2 n4 - 2 n5 - n6 - 2 n7 - 2 n8 - 2 z1 - z2 - z3 - z4 - z5 - z6]
    Gamma[n4 + n5 + z1 + z3 + z4 + z5 + z6]
    Gamma[-4 + 2 eps + n1 + n3 + n4 + n5 + n7 + n8 + 2 z1 + z10 + z12 - z13 + z2 + z3 + z4 + z5 + z6]
    Gamma[12 - 6 eps - 2 n1 - n2 - n3 - 2 n4 - 2 n5 - n6 -
      2 n7 - 2 n8 - z1 + z11 + z12 + z13 - z2 - z3 - z4 - z5 - z7])}
```

fin = BarnesLemma[#, 1] & /@ res

```
>> Barnes 1st Lemma will be checked for:
 {z14, z13, z12, z11, z10, z9, z8, z7, z6, z5, z4, z3, z2, z1} <<
   Starting with dim=14 representation...


1. Checking z14...Barnes Lemma was applied.
2. Checking z13
3. Checking z12
4. Checking z11
5. Checking z10
6. Checking z9
7. Checking z8...Barnes Lemma was applied.
8. Checking z7
9. Checking z6
10. Checking z5
11. Checking z4
12. Checking z3
13. Checking z2
14. Checking z1
>> Representation after 1st Barnes Lemma:  <<
   1st Barnes Lemma was applied for: {z8, z14}
   Obtained representation has: dim=12
```

$$\left\{\left((-1)^{n1+n2+n3+n4+n5+n6+n7+n8}\ (-s)^{\frac{3}{2}(4-2\,eps)-n1-n2-n3-n4-n5-n6-n7-n8}\right.\right.$$
$$\Gamma[-z1]\,\Gamma[-z11]\,\Gamma[-z12]\,\Gamma[-z13]\,\Gamma[-z2]$$
$$\Gamma[6-3\,eps-n1-n2-n3-n4-n5-n7-n8-z1-z2-z3]\,\Gamma[-z3]\,\Gamma[n5+z3]$$
$$\Gamma[-z4]\,\Gamma[-z5]\,\Gamma[6-3\,eps-n1-n3-n4-n5-n6-n7-n8-z1-z4-z5-z6]$$
$$\Gamma[2-eps-n1-n4-n5-z1-z10-z12-z4-z5-z6]\,\Gamma[12-6\,eps-2\,n1-$$
$$n2-2\,n3-2\,n4-2\,n5-n6-2\,n7-2\,n8-2\,z1+z11+z13-z2-z3-z4-z5-z6]$$
$$\Gamma[-z6]\,\Gamma[n3+z1+z12+z2+z6]\,\Gamma[n4+z1+z4+z5+z6]$$
$$\Gamma[-4+2\,eps+n1+n3+n4+n5+n7+n8+2\,z1+z10+z12+z2+z3+z4+z5+z6]\,\Gamma[$$
$$8-4\,eps-n1-n2-n3-n4-n5-n6-2\,n7-n8-z1+z11+z12+z13-z2-z3-z4-z7]$$
$$\Gamma[-z7]\,\Gamma[-8+4\,eps+n1+n2+n3+n4+n5+n6+2\,n7+$$
$$n8+z1-z10-z11-z12-z13+z2+z3+z4+z7]\,\Gamma[$$
$$10-5\,eps-2\,n1-n2-n3-n4-n5-n6-2\,n7-2\,n8-z1+z13-z2-z3-z4-z5-z7-z9]$$
$$\Gamma[12-6\,eps-3\,n1-n2-n3-2\,n4-2\,n5-n6-2\,n7-2\,n8-2\,z1-z10+z13-z2-z3-$$
$$2\,z4-2\,z5-z6-z7-z9]\,\Gamma[-z9]\,\Gamma[n1+z1+z10+z11+z12+z4+z5+z6+z9]$$
$$\Gamma[-6+3\,eps+n1+n2+n3+n4+n5+n6+n7+n8+z1+z2+z3+z4+z5+z6+z7+z9]$$
$$\Gamma[-10+5\,eps+2\,n1+n2+n3+2\,n4+2\,n5+n6+2\,n7+$$
$$\left.2\,n8+2\,z1+z10-z13+z2+2\,z3+2\,z4+2\,z5+z6+z7+z9]\right)\Big/$$
$$(\Gamma[n1]\,\Gamma[n2]\,\Gamma[n3]\,\Gamma[n4]\,\Gamma[n5]\,\Gamma[n6]\,\Gamma[n7]$$
$$\Gamma[8-4\,eps-n1-n2-n3-n4-n5-n6-n7-n8]\,\Gamma[n8]$$
$$\Gamma[12-6\,eps-2\,n1-n2-2\,n3-2\,n4-2\,n5-n6-2\,n7-2\,n8-2\,z1-z2-z3-z4-z5-z6]$$
$$\Gamma[n4+n5+z1+z3+z4+z5+z6]\,\Gamma[12-6\,eps-2\,n1-n2-n3-$$
$$2\,n4-2\,n5-n6-2\,n7-2\,n8-z1+z11+z12+z13-z2-z3-z4-z5-z7]$$
$$\Gamma[12-6\,eps-3\,n1-n2-n3-2\,n4-2\,n5-n6-2\,n7-2\,n8-2\,z1-z10-z12+$$
$$z13-z2-z3-2\,z4-2\,z5-z6-z7-z9]\,\Gamma[-10+5\,eps+2\,n1+n2+2\,n3+2\,n4+$$
$$\left.\left.2\,n5+n6+2\,n7+2\,n8+3\,z1+z10+z12-z13+2\,z2+2\,z3+2\,z4+2\,z5+2\,z6+z7+z9])\right\}$$

```
finres = fin /. {n1 → 1, n2 → 1, n3 → 1, n4 → 1, n5 → 1, n6 → 1, n7 → 1, n8 → 1}
```

```
{((-s)^(-8+3/2 (4-2 eps)) Gamma[-z1] Gamma[-z11] Gamma[-z12] Gamma[-z13] Gamma[-z2]
    Gamma[-1-3 eps-z1-z2-z3] Gamma[-z3] Gamma[1+z3] Gamma[-z4] Gamma[-z5]
    Gamma[-1-3 eps-z1-z4-z5-z6] Gamma[-1-eps-z1-z10-z12-z4-z5-z6]
    Gamma[-2-6 eps-2 z1+z11+z13-z2-z3-z4-z5-z6]
    Gamma[-z6] Gamma[1+z1+z12+z2+z6] Gamma[1+z1+z4+z5+z6]
    Gamma[2+2 eps+2 z1+z10+z12+z2+z3+z4+z5+z6]
    Gamma[-1-4 eps-z1+z11+z12+z13-z2-z3-z4-z7] Gamma[-z7]
    Gamma[1+4 eps+z1-z10-z11-z12-z13+z2+z3+z4+z7]
    Gamma[-1-5 eps-z1+z13-z2-z3-z4-z5-z7-z9]
    Gamma[-2-6 eps-2 z1-z10+z13-z2-z3-2 z4-2 z5-z6-z7-z9]
    Gamma[-z9] Gamma[1+z1+z10+z11+z12+z4+z5+z6+z9]
    Gamma[2+3 eps+z1+z2+z3+z4+z5+z6+z7+z9]
    Gamma[3+5 eps+2 z1+z10-z13+z2+2 z3+2 z4+2 z5+z6+z7+z9])/
  (Gamma[-4 eps] Gamma[-2-6 eps-2 z1-z2-z3-z4-z5-z6] Gamma[
     2+z1+z3+z4+z5+z6] Gamma[-1-6 eps-z1+z11+z12+z13-z2-z3-z4-z5-z7]
    Gamma[-2-6 eps-2 z1-z10-z12+z13-z2-z3-2 z4-2 z5-z6-z7-z9]
    Gamma[4+5 eps+3 z1+z10+z12-z13+2 z2+2 z3+2 z4+2 z5+2 z6+z7+z9])}
```

```
<< MB.m
```

```
MB 1.2

by Michal Czakon

improvements by Alexander Smirnov

more info in hep-ph/0511200

last modified 2 Jan 09
```

```
<< MBresolve.m
```

```
MBresolve 1.0

by Alexander Smirnov

more info in arXiv:0901.0386

last modified 4 Jan 09
```

```
step1 = MBresolve[#, eps] & /@ finres // Flatten;
```

```
CREATING RESIDUES LIST..........336.3197 seconds
EVALUATING RESIDUES..........0.8726 seconds
```

```
step2 = MBexpand[step1, Exp[3 * eps * EulerGamma], {eps, 0, 0}];
MBanalytic = MBmerge[step2]
```

A very large output was generated. Here is a sample of it:

$$\Big\{\text{MBint}\Big[-\frac{1}{1620\,\text{eps}^4\,s^2}\,\text{Gamma}[-z3]^2\,\text{Gamma}[z3-z5]\,\text{Gamma}[-z5]\,\text{Gamma}[1+z5]\,\text{Gamma}[1+z3+z5]$$
$$(360 + 2520\,\text{eps}\,\text{EulerGamma} + 8820\,\text{eps}^2\,\text{EulerGamma}^2 + 20\,580\,\text{eps}^3\,\text{EulerGamma}^3 +$$
$$36\,015\,\text{eps}^4\,\text{EulerGamma}^4 + \ll 264 \gg + 1215\,\text{eps}^4\,\text{PolyGamma}[3, z3-z5] -$$
$$975\,\text{eps}^4\,\text{PolyGamma}[3,-z5] + 2430\,\text{eps}^4\,\text{PolyGamma}[3, 1+z5]),$$
$$\{\{\text{eps}\to 0\}, \{z3\to -0.277981, z5\to -0.350341\}\}\Big], \ll 92 \gg,$$
$$\text{MBint}\Big[(4\,\text{Gamma}[-z11]\,\text{Gamma}[-z10-z12]\ \ll 17 \gg\ \text{Gamma}[z10+z11+z12+z9]$$
$$\text{Gamma}[1+z2+z3+z7+z9]\,\text{Gamma}[1+z10-z13+z2+2\,z3-z6+z7+z9])\Big/$$
$$(3\,s^2\,\text{Gamma}[z11+z12+\ll 5 \gg + z6 - z7]\,\text{Gamma}[-z10-z12+z13-z2-z3+z6-z7-z9]$$
$$\text{Gamma}[1+z10+z12-z13+2\,z2+2\,z3-z4-z5-z6+z7+z9]), \{\ll 1 \gg\}\Big]\Big\}$$

Show Less | Show More | Show Full Output | Set Size Limit...

```
MBintegrate[MBanalytic, {s → -1}, MaxPoints → 10^7]
```

```
Shifting contours...

Performing 0 lower-dimensional integrations with NIntegrate

Higher-dimensional integrals

Preparing MBpart1eps0 (dim 11)

Preparing MBpart2eps0 (dim 10)

Preparing MBpart3eps0 (dim 10)

Preparing MBpart4eps0 (dim 9)

Preparing MBpart5eps0 (dim 9)

Preparing MBpart6eps0 (dim 9)

Preparing MBpart7eps0 (dim 9)

Preparing MBpart8eps0 (dim 9)

Preparing MBpart9eps0 (dim 8)

Preparing MBpart10eps0 (dim 8)

Preparing MBpart11eps0 (dim 8)

Preparing MBpart12eps0 (dim 8)

Preparing MBpart13eps0 (dim 8)

Preparing MBpart14eps0 (dim 8)

Preparing MBpart15eps0 (dim 8)

Preparing MBpart16eps0 (dim 8)

Preparing MBpart17eps0 (dim 8)
```

```
Preparing MBpart18eps0 (dim 7)
Preparing MBpart19eps0 (dim 7)
Preparing MBpart20eps0 (dim 7)
Preparing MBpart21eps0 (dim 7)
Preparing MBpart22eps0 (dim 7)
Preparing MBpart23eps0 (dim 7)
Preparing MBpart24eps0 (dim 7)
Preparing MBpart25eps0 (dim 7)
Preparing MBpart26eps0 (dim 7)
Preparing MBpart27eps0 (dim 7)
Preparing MBpart28eps0 (dim 7)
Preparing MBpart29eps0 (dim 7)
Preparing MBpart30eps0 (dim 7)
Preparing MBpart31eps0 (dim 6)
Preparing MBpart32eps0 (dim 6)
Preparing MBpart33eps0 (dim 6)
Preparing MBpart34eps0 (dim 6)
Preparing MBpart35eps0 (dim 6)
Preparing MBpart36eps0 (dim 6)
Preparing MBpart37eps0 (dim 6)
Preparing MBpart38eps0 (dim 6)
Preparing MBpart39eps0 (dim 6)
Preparing MBpart40eps0 (dim 6)
Preparing MBpart41eps0 (dim 6)
Preparing MBpart42eps0 (dim 6)
Preparing MBpart43eps0 (dim 6)
Preparing MBpart44eps0 (dim 6)
Preparing MBpart45eps0 (dim 6)
Preparing MBpart46eps0 (dim 6)
Preparing MBpart47eps0 (dim 6)
Preparing MBpart48eps0 (dim 6)
Preparing MBpart49eps0 (dim 6)
Preparing MBpart50eps0 (dim 5)
Preparing MBpart51eps0 (dim 5)
Preparing MBpart52eps0 (dim 5)
Preparing MBpart53eps0 (dim 5)
```

```
Preparing MBpart54eps0 (dim 5)
Preparing MBpart55eps0 (dim 5)
Preparing MBpart56eps0 (dim 5)
Preparing MBpart57eps0 (dim 5)
Preparing MBpart58eps0 (dim 5)
Preparing MBpart59eps0 (dim 5)
Preparing MBpart60eps0 (dim 5)
Preparing MBpart61eps0 (dim 5)
Preparing MBpart62eps0 (dim 5)
Preparing MBpart63eps0 (dim 5)
Preparing MBpart64eps0 (dim 5)
Preparing MBpart65eps0 (dim 5)
Preparing MBpart66eps0 (dim 5)
Preparing MBpart67eps0 (dim 5)
Preparing MBpart68eps0 (dim 5)
Preparing MBpart69eps0 (dim 5)
Preparing MBpart70eps0 (dim 4)
Preparing MBpart71eps0 (dim 4)
Preparing MBpart72eps0 (dim 4)
Preparing MBpart73eps0 (dim 4)
Preparing MBpart74eps0 (dim 4)
Preparing MBpart75eps0 (dim 4)
Preparing MBpart76eps0 (dim 4)
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Preparing MBpart78eps0 (dim 4)
Preparing MBpart79eps0 (dim 4)
Preparing MBpart80eps0 (dim 4)
Preparing MBpart81eps0 (dim 4)
Preparing MBpart82eps0 (dim 4)
Preparing MBpart83eps0 (dim 4)
Preparing MBpart84eps0 (dim 4)
Preparing MBpart85eps0 (dim 3)
Preparing MBpart86eps0 (dim 3)
Preparing MBpart87eps0 (dim 3)
Preparing MBpart88eps0 (dim 3)
Preparing MBpart89eps0 (dim 3)
```

```
Preparing MBpart90eps0 (dim 3)
Preparing MBpart91eps0 (dim 3)
Preparing MBpart92eps0 (dim 3)
Preparing MBpart93eps0 (dim 2)
Preparing MBpart94eps0 (dim 2)
Preparing MBpart95eps-1 (dim 8)
Preparing MBpart96eps-1 (dim 8)
Preparing MBpart97eps-1 (dim 7)
Preparing MBpart98eps-1 (dim 7)
Preparing MBpart99eps-1 (dim 7)
Preparing MBpart100eps-1 (dim 7)
Preparing MBpart101eps-1 (dim 7)
Preparing MBpart102eps-1 (dim 6)
Preparing MBpart103eps-1 (dim 6)
Preparing MBpart104eps-1 (dim 6)
Preparing MBpart105eps-1 (dim 6)
Preparing MBpart106eps-1 (dim 6)
Preparing MBpart107eps-1 (dim 6)
Preparing MBpart108eps-1 (dim 6)
Preparing MBpart109eps-1 (dim 6)
Preparing MBpart110eps-1 (dim 6)
Preparing MBpart111eps-1 (dim 6)
Preparing MBpart112eps-1 (dim 6)
Preparing MBpart113eps-1 (dim 6)
Preparing MBpart114eps-1 (dim 5)
Preparing MBpart115eps-1 (dim 5)
Preparing MBpart116eps-1 (dim 5)
Preparing MBpart117eps-1 (dim 5)
Preparing MBpart118eps-1 (dim 5)
Preparing MBpart119eps-1 (dim 5)
Preparing MBpart120eps-1 (dim 5)
Preparing MBpart121eps-1 (dim 5)
Preparing MBpart122eps-1 (dim 5)
Preparing MBpart123eps-1 (dim 5)
Preparing MBpart124eps-1 (dim 5)
Preparing MBpart125eps-1 (dim 5)
```

```
Preparing MBpart126eps-1 (dim 5)
Preparing MBpart127eps-1 (dim 5)
Preparing MBpart128eps-1 (dim 5)
Preparing MBpart129eps-1 (dim 5)
Preparing MBpart130eps-1 (dim 4)
Preparing MBpart131eps-1 (dim 4)
Preparing MBpart132eps-1 (dim 4)
Preparing MBpart133eps-1 (dim 4)
Preparing MBpart134eps-1 (dim 4)
Preparing MBpart135eps-1 (dim 4)
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Preparing MBpart137eps-1 (dim 4)
Preparing MBpart138eps-1 (dim 4)
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Preparing MBpart141eps-1 (dim 4)
Preparing MBpart142eps-1 (dim 4)
Preparing MBpart143eps-1 (dim 4)
Preparing MBpart144eps-1 (dim 3)
Preparing MBpart145eps-1 (dim 3)
Preparing MBpart146eps-1 (dim 3)
Preparing MBpart147eps-1 (dim 3)
Preparing MBpart148eps-1 (dim 3)
Preparing MBpart149eps-1 (dim 3)
Preparing MBpart150eps-1 (dim 3)
Preparing MBpart151eps-1 (dim 3)
Preparing MBpart152eps-1 (dim 2)
Preparing MBpart153eps-1 (dim 2)
Preparing MBpart154eps-2 (dim 6)
Preparing MBpart155eps-2 (dim 6)
Preparing MBpart156eps-2 (dim 5)
Preparing MBpart157eps-2 (dim 5)
Preparing MBpart158eps-2 (dim 5)
Preparing MBpart159eps-2 (dim 5)
Preparing MBpart160eps-2 (dim 5)
Preparing MBpart161eps-2 (dim 5)
```

```
Preparing MBpart162eps-2 (dim 5)
Preparing MBpart163eps-2 (dim 5)
Preparing MBpart164eps-2 (dim 5)
Preparing MBpart165eps-2 (dim 5)
Preparing MBpart166eps-2 (dim 5)
Preparing MBpart167eps-2 (dim 4)
Preparing MBpart168eps-2 (dim 4)
Preparing MBpart169eps-2 (dim 4)
Preparing MBpart170eps-2 (dim 4)
Preparing MBpart171eps-2 (dim 4)
Preparing MBpart172eps-2 (dim 4)
Preparing MBpart173eps-2 (dim 4)
Preparing MBpart174eps-2 (dim 4)
Preparing MBpart175eps-2 (dim 4)
Preparing MBpart176eps-2 (dim 4)
Preparing MBpart177eps-2 (dim 4)
Preparing MBpart178eps-2 (dim 4)
Preparing MBpart179eps-2 (dim 3)
Preparing MBpart180eps-2 (dim 3)
Preparing MBpart181eps-2 (dim 3)
Preparing MBpart182eps-2 (dim 3)
Preparing MBpart183eps-2 (dim 3)
Preparing MBpart184eps-2 (dim 3)
Preparing MBpart185eps-2 (dim 3)
Preparing MBpart186eps-2 (dim 3)
Preparing MBpart187eps-2 (dim 2)
Preparing MBpart188eps-2 (dim 2)
Preparing MBpart189eps-3 (dim 4)
Preparing MBpart190eps-3 (dim 4)
Preparing MBpart191eps-3 (dim 4)
Preparing MBpart192eps-3 (dim 4)
Preparing MBpart193eps-3 (dim 4)
Preparing MBpart194eps-3 (dim 4)
Preparing MBpart195eps-3 (dim 4)
Preparing MBpart196eps-3 (dim 3)
Preparing MBpart197eps-3 (dim 3)
```

```
Preparing MBpart198eps-3 (dim 3)
Preparing MBpart199eps-3 (dim 3)
Preparing MBpart200eps-3 (dim 3)
Preparing MBpart201eps-3 (dim 3)
Preparing MBpart202eps-3 (dim 3)
Preparing MBpart203eps-3 (dim 2)
Preparing MBpart204eps-3 (dim 2)
Preparing MBpart205eps-4 (dim 3)
Preparing MBpart206eps-4 (dim 3)
Preparing MBpart207eps-4 (dim 3)
Preparing MBpart208eps-4 (dim 2)
Preparing MBpart209eps-4 (dim 2)
Running MBpart1eps0
Running MBpart2eps0
Running MBpart3eps0
Running MBpart4eps0
Running MBpart5eps0
Running MBpart6eps0
Running MBpart7eps0
Running MBpart8eps0
Running MBpart9eps0
Running MBpart10eps0
Running MBpart11eps0
Running MBpart12eps0
Running MBpart13eps0
Running MBpart14eps0
Running MBpart15eps0
Running MBpart16eps0
Running MBpart17eps0
Running MBpart18eps0
Running MBpart19eps0
Running MBpart20eps0
Running MBpart21eps0
Running MBpart22eps0
Running MBpart23eps0
Running MBpart24eps0
```

```
Running MBpart25eps0
Running MBpart26eps0
Running MBpart27eps0
Running MBpart28eps0
Running MBpart29eps0
Running MBpart30eps0
Running MBpart31eps0
Running MBpart32eps0
Running MBpart33eps0
Running MBpart34eps0
Running MBpart35eps0
Running MBpart36eps0
Running MBpart37eps0
Running MBpart38eps0
Running MBpart39eps0
Running MBpart40eps0
Running MBpart41eps0
Running MBpart42eps0
Running MBpart43eps0
Running MBpart44eps0
Running MBpart45eps0
Running MBpart46eps0
Running MBpart47eps0
Running MBpart48eps0
Running MBpart49eps0
Running MBpart50eps0
Running MBpart51eps0
Running MBpart52eps0
Running MBpart53eps0
Running MBpart54eps0
Running MBpart55eps0
Running MBpart56eps0
Running MBpart57eps0
Running MBpart58eps0
Running MBpart59eps0
Running MBpart60eps0
```

```
Running MBpart61eps0
Running MBpart62eps0
Running MBpart63eps0
Running MBpart64eps0
Running MBpart65eps0
Running MBpart66eps0
Running MBpart67eps0
Running MBpart68eps0
Running MBpart69eps0
Running MBpart70eps0
Running MBpart71eps0
Running MBpart72eps0
Running MBpart73eps0
Running MBpart74eps0
Running MBpart75eps0
Running MBpart76eps0
Running MBpart77eps0
Running MBpart78eps0
Running MBpart79eps0
Running MBpart80eps0
Running MBpart81eps0
Running MBpart82eps0
Running MBpart83eps0
Running MBpart84eps0
Running MBpart85eps0
Running MBpart86eps0
Running MBpart87eps0
Running MBpart88eps0
Running MBpart89eps0
Running MBpart90eps0
Running MBpart91eps0
Running MBpart92eps0
Running MBpart93eps0
Running MBpart94eps0
Running MBpart95eps-1
Running MBpart96eps-1
```

```
Running MBpart97eps-1
Running MBpart98eps-1
Running MBpart99eps-1
Running MBpart100eps-1
Running MBpart101eps-1
Running MBpart102eps-1
Running MBpart103eps-1
Running MBpart104eps-1
Running MBpart105eps-1
Running MBpart106eps-1
Running MBpart107eps-1
Running MBpart108eps-1
Running MBpart109eps-1
Running MBpart110eps-1
Running MBpart111eps-1
Running MBpart112eps-1
Running MBpart113eps-1
Running MBpart114eps-1
Running MBpart115eps-1
Running MBpart116eps-1
Running MBpart117eps-1
Running MBpart118eps-1
Running MBpart119eps-1
Running MBpart120eps-1
Running MBpart121eps-1
Running MBpart122eps-1
Running MBpart123eps-1
Running MBpart124eps-1
Running MBpart125eps-1
Running MBpart126eps-1
Running MBpart127eps-1
Running MBpart128eps-1
Running MBpart129eps-1
Running MBpart130eps-1
Running MBpart131eps-1
Running MBpart132eps-1
```

```
Running MBpart133eps-1
Running MBpart134eps-1
Running MBpart135eps-1
Running MBpart136eps-1
Running MBpart137eps-1
Running MBpart138eps-1
Running MBpart139eps-1
Running MBpart140eps-1
Running MBpart141eps-1
Running MBpart142eps-1
Running MBpart143eps-1
Running MBpart144eps-1
Running MBpart145eps-1
Running MBpart146eps-1
Running MBpart147eps-1
Running MBpart148eps-1
Running MBpart149eps-1
Running MBpart150eps-1
Running MBpart151eps-1
Running MBpart152eps-1
Running MBpart153eps-1
Running MBpart154eps-2
Running MBpart155eps-2
Running MBpart156eps-2
Running MBpart157eps-2
Running MBpart158eps-2
Running MBpart159eps-2
Running MBpart160eps-2
Running MBpart161eps-2
Running MBpart162eps-2
Running MBpart163eps-2
Running MBpart164eps-2
Running MBpart165eps-2
Running MBpart166eps-2
Running MBpart167eps-2
Running MBpart168eps-2
```

```
Running MBpart169eps-2
Running MBpart170eps-2
Running MBpart171eps-2
Running MBpart172eps-2
Running MBpart173eps-2
Running MBpart174eps-2
Running MBpart175eps-2
Running MBpart176eps-2
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Running MBpart193eps-3
Running MBpart194eps-3
Running MBpart195eps-3
Running MBpart196eps-3
Running MBpart197eps-3
Running MBpart198eps-3
Running MBpart199eps-3
Running MBpart200eps-3
Running MBpart201eps-3
Running MBpart202eps-3
Running MBpart203eps-3
Running MBpart204eps-3
```

```
Running MBpart205eps-4

Running MBpart206eps-4

Running MBpart207eps-4

Running MBpart208eps-4

Running MBpart209eps-4
```

$$\left\{-15.9738+\frac{0.274193}{\text{eps}^4}-\frac{0.80132}{\text{eps}^3}-\frac{8.11761}{\text{eps}^2}-\frac{26.1734}{\text{eps}},\right.$$
$$\left.\left\{1.62908+\frac{0.0000787849}{\text{eps}^4}+\frac{0.000498811}{\text{eps}^3}+\frac{0.00803399}{\text{eps}^2}+\frac{0.0896681}{\text{eps}},\ 0\right\}\right\}$$

```
Quit[]

<< ../../FIESTA3.2/FIESTA3.m

UsingQLink = False; UsingC = False;

CurrentIntegratorOptions = {{"maxeval", "5000000"}};
```

```
SDEvaluate[UF[{k1, k2, k3},
  {-k1^2, -k2^2, -k3^2, -(k2 - k1)^2, -(k2 - k1 - p1)^2, -(k2 - p1 - p2)^2,
   -(k2 - k3 - p1 - p2)^2, -(k2 - k1 - k3 - p1)^2}, {p1^2 → 0, p2^2 → 0, p1 * p2 → s / 2,
   s → -1}], {1, 1, 1, 1, 1, 1, 1, 1}, 0]
```

```
FIESTA 3.2
Starting 1 subkernels
Subkernel will be used for launching external programs, all evaluations go on main kernel.
UsingC: False
NumberOfLinks: 1
UsingQLink: False
Strategy: STRATEGY_S
Integration has to be performed up to order 0
Sector decomposition - 8 sectors
Primary sector 1 resulted in 187 sectors.
Primary sector 2 resulted in 120 sectors.
Primary sector 3 resulted in 84 sectors.
Primary sector 4 resulted in 104 sectors.
Primary sector 5 resulted in 101 sectors.
Primary sector 6 resulted in 89 sectors.
Primary sector 7 resulted in 158 sectors.
Primary sector 8 resulted in 197 sectors.
Totally: 16.5294 seconds; 16 sectors.
Preparing database: 0.0375 seconds.
Variable substitution..........4.1044 seconds; 1040 terms.
Pole resolution..........3.8202 seconds; 3695 terms.
Expression preparation..........3.4054 seconds; 3695 terms.
Epsilon expansion..........5.1994 seconds; 7343 terms.
Preparing integration strings..........16.6828 seconds; 7323 terms.
Database ready for integration.
Terms of order -4: 30, max vars: 3
Integrating..........10.288 seconds.
Returned answer: 0.27414597932553747 + pm* 4.030493971520338*^-6
(0.2741459793255376 + 4.030493971520339*^-6*pm1)*ep^(-4)
Terms of order -3: 259, max vars: 4
Integrating..........71.9194 seconds.
Returned answer: -1.6243017980232157 + pm* 0
(0.2741459793255376 + 4.030493971520339*^-6*pm2)*ep^(-4)+(-0.8018638600466034 + 0.00001209
Terms of order -2: 975, max vars: 5
Integrating..........238.0287 seconds.
Returned answer: -5.29397427335636 + pm* 0
(0.2741459793255376 + 4.030493971520339*^-6*pm4)*ep^(-4)+(-0.8018638600466034 + 0.00001209
Terms of order -1: 2370, max vars: 6
Integrating..........517.141 seconds.
Returned answer: -1.4807054287931052 + pm* 0
(0.2741459793255376 + 4.030493971520339*^-6*pm7)*ep^(-4)+(-0.8018638600466034 + 0.00001209
Terms of order 0: 3689, max vars: 7
Integrating..........873.7327 seconds.
Returned answer: 43.351816874115734 + pm* 0
(0.2741459793255376 + 4.030493971520339*^-6*pm11)*ep^(-4)+(-0.8018638600466034 + 0.0000120
Total integration time: 1711.1268
Total time used: 1761.5 seconds.
```

$$-14.1536 + \frac{0.274146 + 4.03049 \times 10^{-6}\,\text{pm16}}{\text{ep}^4} + \frac{-0.801864 + 0.0000120915\,\text{pm17}}{\text{ep}^3} +$$
$$\frac{-8.1376 + 0.0000298345\,\text{pm18}}{\text{ep}^2} + \frac{-26.264 + 0.0000458997\,\text{pm19}}{\text{ep}} + 0.0000679453\,\text{pm20}$$

```
Quit[]
```

Example#4, limitations 1

Present version cannot generate representation for planar diagrams and non-planar diagrams with a planar subloop like in this example (this possibility will be added in a future release)

```
<< AMBREv3.1.m
```

```
AMBRE v3.1 [Jul 2016] by I.Dubovyk, http://us.edu.pl/~gluza/ambre/
License: http://creativecommons.org, CC BY-ND
Ref.: I. Dubovyk, J. Gluza, T. Riemann, J. Usovitsch, arXiv:1607.07538.
```

```
<< PlanarityTestv1.2.m
```

```
by E. Dubovyk and K. Bielas ver: 1.2
created: January 2014
last executed: 25.07.2016 at 15:39
```

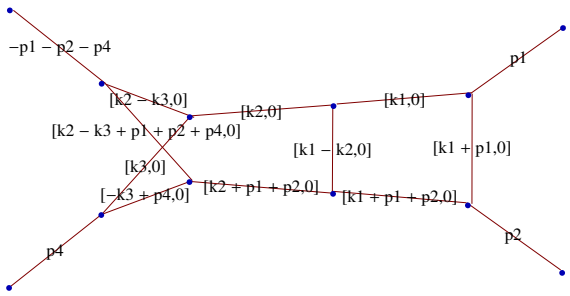
```
invariants = {p1^2 → 0, p2^2 → 0, p3^2 → 0, p4^2 → 0, p1 * p2 → 1 / 2 * s, p3 * p4 → 1 / 2 s,
    p1 * p3 → 1 / 2 * t, p2 * p4 → 1 / 2 * t, p2 * p3 → 1 / 2 * u, p1 * p4 → 1 / 2 * u};

ex3 = PR[k1, 0, n1] PR[k1 + p1, 0, n2] PR[k1 + p1 + p2, 0, n3]
    PR[k1 - k2, 0, n4] PR[k2, 0, n5] PR[k2 + p1 + p2, 0, n6]
    PR[p1 + p2 + p4 + k2 - k3, 0, n7] PR[k2 - k3, 0, n8] PR[k3, 0, n9] PR[p4 - k3, 0, n10];

PlanarityTest[{ex3}, {k1, k2, k3}, DrawGraph → True];
```

```
The Diagram

is non-planar.
```




```
res = MBreprNP[{1}, {ex3}, {k1, k2, k3}]
```

Fauto::mode : F polynomial will be calculated in AUTO mode. In order to use MANUAL mode execute Fauto[0].

```
Upoly = x[1] x[2] x[4] + x[1] x[3] x[4] + x[2] x[3] x[4] + x[1] x[2] x[5] + x[1] x[3] x[5] +
  x[2] x[3] x[5] + x[1] x[4] x[5] + x[2] x[4] x[5] + x[2] x[4] x[6] + x[3] x[4] x[6] +
  x[2] x[5] x[6] + x[3] x[5] x[6] + x[4] x[5] x[6] + x[2] x[4] x[7] + x[3] x[4] x[7] +
  x[2] x[5] x[7] + x[3] x[5] x[7] + x[4] x[5] x[7] + x[1] x[4] x[8] + x[2] x[4] x[8] +
  x[1] x[5] x[8] + x[2] x[5] x[8] + x[4] x[6] x[8] + x[5] x[6] x[8] + x[4] x[7] x[8] +
  x[5] x[7] x[8] + x[1] x[2] x[9] + x[1] x[3] x[9] + x[2] x[3] x[9] + x[1] x[4] x[9] +
  x[2] x[4] x[9] + x[2] x[6] x[9] + x[3] x[6] x[9] + x[4] x[6] x[9] + x[2] x[7] x[9] +
  x[3] x[7] x[9] + x[4] x[7] x[9] + x[1] x[8] x[9] + x[2] x[8] x[9] + x[6] x[8] x[9] +
  x[7] x[8] x[9] + x[1] x[2] x[10] + x[1] x[3] x[10] + x[2] x[3] x[10] + x[1] x[5] x[10] +
  x[2] x[5] x[10] + x[2] x[6] x[10] + x[3] x[6] x[10] + x[5] x[6] x[10] + x[2] x[7] x[10] +
  x[3] x[7] x[10] + x[5] x[7] x[10] + x[1] x[8] x[10] + x[2] x[8] x[10] + x[6] x[8] x[10] +
  x[7] x[8] x[10] + x[1] x[9] x[10] + x[2] x[9] x[10] + x[6] x[9] x[10] + x[7] x[9] x[10]

Fpoly = -s x[1] x[2] x[4] x[7] - s x[1] x[3] x[4] x[7] - s x[2] x[3] x[4] x[7] -
  s x[1] x[2] x[5] x[7] - s x[1] x[3] x[5] x[7] - s x[2] x[3] x[5] x[7] - s x[1] x[4] x[5] x[7] -
  s x[2] x[4] x[5] x[7] - s x[1] x[2] x[4] x[8] - s x[1] x[3] x[4] x[8] - s x[2] x[3] x[4] x[8] -
  s x[1] x[2] x[5] x[8] - s x[1] x[3] x[5] x[8] - s x[2] x[3] x[5] x[8] - s x[1] x[4] x[5] x[8] -
  s x[2] x[4] x[5] x[8] - s x[3] x[4] x[6] x[8] - s x[3] x[5] x[6] x[8] - s x[4] x[5] x[6] x[8] -
  s x[1] x[4] x[7] x[8] - s x[3] x[4] x[7] x[8] - s x[1] x[5] x[7] x[8] - s x[3] x[5] x[7] x[8] -
  s x[4] x[5] x[7] x[8] - p4^2 x[1] x[2] x[4] x[9] - p4^2 x[1] x[3] x[4] x[9] -
  p4^2 x[2] x[3] x[4] x[9] - p4^2 x[1] x[2] x[5] x[9] - p4^2 x[1] x[3] x[5] x[9] -
  p4^2 x[2] x[3] x[5] x[9] - p4^2 x[1] x[4] x[5] x[9] - p4^2 x[2] x[4] x[5] x[9] -
  2 p1 p4 x[2] x[4] x[6] x[9] - p4^2 x[2] x[4] x[6] x[9] - p4^2 x[3] x[4] x[6] x[9] -
  p4^2 x[2] x[5] x[6] x[9] - p4^2 x[3] x[5] x[6] x[9] - p4^2 x[4] x[5] x[6] x[9] -
  s x[1] x[2] x[7] x[9] - s x[1] x[3] x[7] x[9] - s x[2] x[3] x[7] x[9] - s x[1] x[4] x[7] x[9] -
  2 p1 p4 x[2] x[4] x[7] x[9] - 2 p2 p4 x[2] x[4] x[7] x[9] - p4^2 x[2] x[4] x[7] x[9] -
  s x[2] x[4] x[7] x[9] - p4^2 x[3] x[4] x[7] x[9] - p4^2 x[2] x[5] x[7] x[9] -
  p4^2 x[3] x[5] x[7] x[9] - p4^2 x[4] x[5] x[7] x[9] - s x[1] x[2] x[8] x[9] -
  s x[1] x[3] x[8] x[9] - s x[2] x[3] x[8] x[9] - 2 p1 p4 x[1] x[4] x[8] x[9] -
  2 p2 p4 x[1] x[4] x[8] x[9] - p4^2 x[1] x[4] x[8] x[9] - s x[1] x[4] x[8] x[9] -
  2 p1 p4 x[2] x[4] x[8] x[9] - 2 p2 p4 x[2] x[4] x[8] x[9] - p4^2 x[2] x[4] x[8] x[9] -
  s x[2] x[4] x[8] x[9] - p4^2 x[1] x[5] x[8] x[9] - p4^2 x[2] x[5] x[8] x[9] -
  s x[3] x[6] x[8] x[9] - 2 p1 p4 x[4] x[6] x[8] x[9] - 2 p2 p4 x[4] x[6] x[8] x[9] -
  p4^2 x[4] x[6] x[8] x[9] - s x[4] x[6] x[8] x[9] - p4^2 x[5] x[6] x[8] x[9] - s x[1] x[7] x[8] x[9] -
  s x[3] x[7] x[8] x[9] - 2 p1 p4 x[4] x[7] x[8] x[9] - 2 p2 p4 x[4] x[7] x[8] x[9] -
  p4^2 x[4] x[7] x[8] x[9] - s x[4] x[7] x[8] x[9] - p4^2 x[5] x[7] x[8] x[9] -
  2 p1 p4 x[1] x[2] x[4] x[10] - 2 p2 p4 x[1] x[2] x[4] x[10] - p4^2 x[1] x[2] x[4] x[10] -
  s x[1] x[2] x[4] x[10] - 2 p1 p4 x[1] x[3] x[4] x[10] - 2 p2 p4 x[1] x[3] x[4] x[10] -
  p4^2 x[1] x[3] x[4] x[10] - s x[1] x[3] x[4] x[10] - 2 p1 p4 x[2] x[3] x[4] x[10] -
  2 p2 p4 x[2] x[3] x[4] x[10] - p4^2 x[2] x[3] x[4] x[10] - s x[2] x[3] x[4] x[10] -
  2 p1 p4 x[1] x[2] x[5] x[10] - 2 p2 p4 x[1] x[2] x[5] x[10] - p4^2 x[1] x[2] x[5] x[10] -
  s x[1] x[2] x[5] x[10] - 2 p1 p4 x[1] x[3] x[5] x[10] - 2 p2 p4 x[1] x[3] x[5] x[10] -
  p4^2 x[1] x[3] x[5] x[10] - s x[1] x[3] x[5] x[10] - 2 p1 p4 x[2] x[3] x[5] x[10] -
  2 p2 p4 x[2] x[3] x[5] x[10] - p4^2 x[2] x[3] x[5] x[10] - s x[2] x[3] x[5] x[10] -
  2 p1 p4 x[1] x[4] x[5] x[10] - 2 p2 p4 x[1] x[4] x[5] x[10] - p4^2 x[1] x[4] x[5] x[10] -
  s x[1] x[4] x[5] x[10] - 2 p1 p4 x[2] x[4] x[5] x[10] - 2 p2 p4 x[2] x[4] x[5] x[10] -
  p4^2 x[2] x[4] x[5] x[10] - s x[2] x[4] x[5] x[10] - 2 p1 p4 x[2] x[4] x[6] x[10] -
  2 p2 p4 x[2] x[4] x[6] x[10] - p4^2 x[2] x[4] x[6] x[10] - s x[2] x[4] x[6] x[10] -
  2 p1 p4 x[3] x[4] x[6] x[10] - 2 p2 p4 x[3] x[4] x[6] x[10] - p4^2 x[3] x[4] x[6] x[10] -
  s x[3] x[4] x[6] x[10] - 2 p2 p4 x[2] x[5] x[6] x[10] - p4^2 x[2] x[5] x[6] x[10] -
  2 p1 p4 x[3] x[5] x[6] x[10] - 2 p2 p4 x[3] x[5] x[6] x[10] - p4^2 x[3] x[5] x[6] x[10] -
  s x[3] x[5] x[6] x[10] - 2 p1 p4 x[4] x[5] x[6] x[10] - 2 p2 p4 x[4] x[5] x[6] x[10] -
  p4^2 x[4] x[5] x[6] x[10] - s x[4] x[5] x[6] x[10] - s x[1] x[2] x[7] x[10] -
```

```
s x[1] x[3] x[7] x[10] - s x[2] x[3] x[7] x[10] - 2 p1 p4 x[2] x[4] x[7] x[10] -
2 p2 p4 x[2] x[4] x[7] x[10] - p4^2 x[2] x[4] x[7] x[10] - s x[2] x[4] x[7] x[10] -
2 p1 p4 x[3] x[4] x[7] x[10] - 2 p2 p4 x[3] x[4] x[7] x[10] - p4^2 x[3] x[4] x[7] x[10] -
s x[3] x[4] x[7] x[10] - s x[1] x[5] x[7] x[10] - p4^2 x[2] x[5] x[7] x[10] -
2 p1 p4 x[3] x[5] x[7] x[10] - 2 p2 p4 x[3] x[5] x[7] x[10] - p4^2 x[3] x[5] x[7] x[10] -
s x[3] x[5] x[7] x[10] - 2 p1 p4 x[4] x[5] x[7] x[10] - 2 p2 p4 x[4] x[5] x[7] x[10] -
p4^2 x[4] x[5] x[7] x[10] - s x[4] x[5] x[7] x[10] - s x[1] x[2] x[8] x[10] -
s x[1] x[3] x[8] x[10] - s x[2] x[3] x[8] x[10] - 2 p1 p4 x[1] x[4] x[8] x[10] -
2 p2 p4 x[1] x[4] x[8] x[10] - p4^2 x[1] x[4] x[8] x[10] - s x[1] x[4] x[8] x[10] -
2 p1 p4 x[2] x[4] x[8] x[10] - 2 p2 p4 x[2] x[4] x[8] x[10] - p4^2 x[2] x[4] x[8] x[10] -
s x[2] x[4] x[8] x[10] - p4^2 x[1] x[5] x[8] x[10] - p4^2 x[2] x[5] x[8] x[10] -
s x[3] x[6] x[8] x[10] - 2 p1 p4 x[4] x[6] x[8] x[10] - 2 p2 p4 x[4] x[6] x[8] x[10] -
p4^2 x[4] x[6] x[8] x[10] - s x[4] x[6] x[8] x[10] - p4^2 x[5] x[6] x[8] x[10] -
s x[1] x[7] x[8] x[10] - s x[3] x[7] x[8] x[10] - 2 p1 p4 x[4] x[7] x[8] x[10] -
2 p2 p4 x[4] x[7] x[8] x[10] - p4^2 x[4] x[7] x[8] x[10] - s x[4] x[7] x[8] x[10] -
p4^2 x[5] x[7] x[8] x[10] - s x[1] x[2] x[9] x[10] - s x[1] x[3] x[9] x[10] -
s x[2] x[3] x[9] x[10] - 2 p1 p4 x[1] x[4] x[9] x[10] - 2 p2 p4 x[1] x[4] x[9] x[10] -
p4^2 x[1] x[4] x[9] x[10] - s x[1] x[4] x[9] x[10] - 2 p1 p4 x[2] x[4] x[9] x[10] -
2 p2 p4 x[2] x[4] x[9] x[10] - p4^2 x[2] x[4] x[9] x[10] - s x[2] x[4] x[9] x[10] -
p4^2 x[1] x[5] x[9] x[10] - p4^2 x[2] x[5] x[9] x[10] - s x[3] x[6] x[9] x[10] -
2 p1 p4 x[4] x[6] x[9] x[10] - 2 p2 p4 x[4] x[6] x[9] x[10] - p4^2 x[4] x[6] x[9] x[10] -
s x[4] x[6] x[9] x[10] - p4^2 x[5] x[6] x[9] x[10] - s x[1] x[7] x[9] x[10] -
s x[3] x[7] x[9] x[10] - 2 p1 p4 x[4] x[7] x[9] x[10] - 2 p2 p4 x[4] x[7] x[9] x[10] -
p4^2 x[4] x[7] x[9] x[10] - s x[4] x[7] x[9] x[10] - p4^2 x[5] x[7] x[9] x[10]
```

ERROR. input integral is planar (run AMBREv2.1 or older) or contains a planar subgraph and cannot be processed by current version of the program.

$Aborted

Quit[]

Example#5, limitations 2

too high dimensionality of the result
(will be improved in a future release)

<< AMBREv3.1.m

```
AMBRE v3.1 [Jul 2016] by I.Dubovyk, http://us.edu.pl/~gluza/ambre/
License: http://creativecommons.org, CC BY-ND
Ref.: I. Dubovyk, J. Gluza, T. Riemann, J. Usovitsch, arXiv:1607.07538.
```

<< PlanarityTestv1.2.m

```
by E. Dubovyk and K. Bielas ver: 1.2
created: January 2014
last executed: 25.07.2016 at 15:28
```

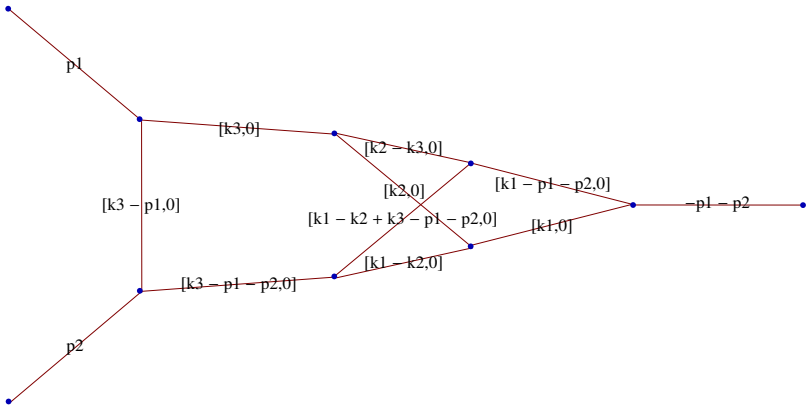
```
invariants = {p1^2 → 0, p2^2 → 0, p1 * p2 → s / 2};

ex5 = PR[k1, 0, n1] PR[k2, 0, n2] PR[k3, 0, n3]
    PR[k1 - k2, 0, n4] PR[k2 - k3, 0, n5] PR[k1 - p1 - p2, 0, n6]
    PR[k1 - k2 + k3 - p1 - p2, 0, n7] PR[k3 - p1, 0, n8] PR[k3 - p1 - p2, 0, n9];
```

```
PlanarityTest[{ex5}, {k1, k2, k3}, DrawGraph → True];
```

The Diagram

is non-planar.




```
res = MBreprNP[{1}, {ex5}, {k1, k2, k3}]
```

Fauto::mode : F polynomial will be calculated in AUTO mode. In order to use MANUAL mode execute Fauto[0].

```
Upoly = x[1] x[2] x[4] + x[1] x[3] x[4] + x[2] x[3] x[4] +
  x[1] x[2] x[5] + x[1] x[3] x[5] + x[2] x[3] x[5] + x[1] x[4] x[5] + x[2] x[4] x[5] +
  x[1] x[2] x[6] + x[1] x[3] x[6] + x[2] x[3] x[6] + x[1] x[4] x[6] + x[2] x[4] x[6] +
  x[2] x[4] x[7] + x[3] x[4] x[7] + x[2] x[5] x[7] + x[3] x[5] x[7] + x[4] x[5] x[7] +
  x[2] x[6] x[7] + x[3] x[6] x[7] + x[4] x[6] x[7] + x[1] x[2] x[8] + x[1] x[3] x[8] +
  x[2] x[3] x[8] + x[1] x[4] x[8] + x[2] x[4] x[8] + x[2] x[7] x[8] + x[3] x[7] x[8] +
  x[4] x[7] x[8] + x[1] x[2] x[9] + x[1] x[3] x[9] + x[2] x[3] x[9] + x[2] x[4] x[9] +
  x[3] x[4] x[9] + x[1] x[5] x[9] + x[3] x[5] x[9] + x[4] x[5] x[9] + x[1] x[6] x[9] +
  x[3] x[6] x[9] + x[4] x[6] x[9] + x[2] x[7] x[9] + x[3] x[7] x[9] + x[5] x[7] x[9] +
  x[6] x[7] x[9] + x[1] x[8] x[9] + x[3] x[8] x[9] + x[4] x[8] x[9] + x[7] x[8] x[9]
```

```
Fpoly = -s x[1] x[2] x[4] x[7] - s x[1] x[3] x[4] x[7] - s x[2] x[3] x[4] x[7] -
  s x[1] x[2] x[5] x[7] - s x[1] x[3] x[5] x[7] - s x[2] x[3] x[5] x[7] - s x[1] x[4] x[5] x[7] -
  s x[2] x[4] x[5] x[7] - s x[1] x[2] x[6] x[7] - s x[1] x[3] x[6] x[7] - s x[2] x[3] x[6] x[7] -
  s x[1] x[4] x[6] x[7] - s x[1] x[2] x[4] x[8] - s x[1] x[3] x[4] x[8] - s x[2] x[3] x[4] x[8] -
  s x[1] x[2] x[5] x[8] - s x[1] x[3] x[5] x[8] - s x[2] x[3] x[5] x[8] - s x[1] x[4] x[5] x[8] -
  s x[2] x[4] x[5] x[8] - s x[1] x[2] x[7] x[8] - s x[1] x[3] x[7] x[8] - s x[2] x[3] x[7] x[8] -
  s x[1] x[4] x[7] x[8] - s x[3] x[4] x[7] x[8] - s x[2] x[5] x[7] x[8] - s x[3] x[5] x[7] x[8] -
  s x[4] x[5] x[7] x[8] - s x[1] x[2] x[4] x[9] - s x[1] x[3] x[4] x[9] - s x[2] x[3] x[4] x[9] -
  s x[1] x[2] x[5] x[9] - s x[1] x[3] x[5] x[9] - s x[2] x[3] x[5] x[9] - s x[1] x[4] x[5] x[9] -
  s x[2] x[4] x[5] x[9] - s x[1] x[4] x[6] x[9] - s x[1] x[2] x[7] x[9] - s x[1] x[3] x[7] x[9] -
  s x[2] x[3] x[7] x[9] - s x[1] x[5] x[7] x[9] - s x[2] x[5] x[7] x[9] - s x[1] x[6] x[7] x[9] -
  s x[1] x[4] x[8] x[9] - s x[3] x[4] x[8] x[9] - s x[1] x[5] x[8] x[9] - s x[3] x[5] x[8] x[9] -
  s x[4] x[5] x[8] x[9] - s x[1] x[7] x[8] x[9] - s x[3] x[7] x[8] x[9] - s x[5] x[7] x[8] x[9]
```

$$\left\{\left((-1)^{n1+n2+n3+n4+n5+n6+n7+n8+n9}\ (-s)^{\frac{3}{2}(4-2\,eps)-n1-n2-n3-n4-n5-n6-n7-n8-n9}\right.\right.$$

$$\text{Gamma}\left[-\frac{3}{2}(4-2\,\text{eps})+n1+n2+n3+n4+n5+n6+n7+n8+n9\right]\text{Gamma}[-z1]$$

$$\text{Gamma}[-z10]\,\text{Gamma}[-z11]\,\text{Gamma}[-z12]\,\text{Gamma}[-z13]\,\text{Gamma}[-z14]$$

```
  Gamma[-z15] Gamma[-z16] Gamma[-z17] Gamma[-z18] Gamma[-z19]
  Gamma[-z2] Gamma[-z20] Gamma[-z21] Gamma[-z22] Gamma[-z23] Gamma[-z24]
  Gamma[-z25] Gamma[-z26] Gamma[-z28] Gamma[-z3] Gamma[-z30] Gamma[-z31]
  Gamma[-z29 + z31] Gamma[-z4] Gamma[-z5] Gamma[-z6] Gamma[-z7]
  Gamma[8 - 4 eps - n1 - n2 - n3 - n4 - 2 n5 - n6 - n7 - n8 - n9 - z1 - z10 - z13 - z16 -
    z2 - z20 - z21 - z22 - z24 + z28 + z29 - z3 + z30 - z4 - z5 - z8] Gamma[-z8]
  Gamma[-8 + 4 eps + n1 + n2 + n3 + n4 + 2 n5 + n6 + n7 + n8 + n9 + z1 + z10 + z13 + z16 +
    z2 + z20 + z21 + z22 + z24 - z27 - z28 - z29 + z3 - z30 + z4 + z5 + z8]
  Gamma[10 - 5 eps - 2 n1 - n2 - n3 - n4 - 2 n5 - 2 n6 - 2 n7 - n8 - n9 - z1 - 2 z10 -
    2 z11 - z12 - 2 z13 - 2 z14 - z15 - z16 - z17 - z19 - 2 z2 - z20 - z21 -
    z22 - z23 - z24 - z26 - 2 z3 + z30 + z31 - 2 z4 - z5 - z6 - 2 z8 - 2 z9]
  Gamma[12 - 6 eps - 2 n1 - 2 n2 - n3 - 2 n4 - 2 n5 - 2 n6 - 2 n7 - n8 - n9 - 2 z1 -
    2 z10 - 2 z11 - z12 - z13 - z14 - z15 - z16 - z17 - z18 - z19 - 2 z2 -
    z20 - z21 - z22 - z23 + z29 - 2 z3 - z4 - 2 z5 - 2 z6 - z7 - 2 z8 - 2 z9]
  Gamma[2 - eps - n1 - n4 - n6 - z10 - z11 - 2 z12 - z13 - z14 - z15 - z17 -
    z2 - z20 - z22 - z25 - z27 - z31 - z5 - z6 - z7 - 2 z8 - 2 z9]
  Gamma[6 - 3 eps - n1 - n3 - n4 - n5 - n6 - n7 - n8 - n9 - z12 - z13 - z14 - z16 -
    z17 - z2 - z20 - z22 - z24 - z25 - z26 + z28 + z30 - z4 - z8 - z9]
  Gamma[6 - 3 eps - n1 - n2 - n3 - n4 - n5 - n6 - n7 - n8 - z1 - z10 - z11 - z12 - z13 -
    z14 - z15 - z16 - z17 - z18 - z19 - z2 - z20 - z3 - z4 - z5 - z6 - z7 - z8 - z9]
  Gamma[6 - 3 eps - n1 - n2 - n4 - n5 - n6 - n7 - n8 - n9 - z1 - z10 - z11 - z12 -
    z13 - z14 - z15 - z2 - z21 - z22 - z23 - z3 - z4 - z5 - z6 - z7 - z8 - z9]
  Gamma[-z9] Gamma[2 - eps - n2 + z12 + z13 + z14 + z16 + z17 + z2 +
    z20 + z22 + z24 + z25 + z26 + z27 + z29 + z4 + z8 + z9]
  Gamma[n1 + z10 + z11 + z12 + z13 + z14 + z15 + z19 + z2 + z22 + z3 + z4 + z8 + z9]
  Gamma[n6 + z10 + z11 + z12 + z13 + z14 + z15 + z20 + z23 + z5 + z6 + z7 + z8 + z9]
  Gamma[-6 + 3 eps + n1 + n2 + n3 + n4 + n5 + n6 + n7 + n8 + n9 + z1 +
    z10 + z11 + z12 + z13 + z14 + z15 + z16 + z17 + z18 + z19 + z2 + z20 +
    z21 + z22 + z23 + z24 + z25 + z26 + z3 + z4 + z5 + z6 + z7 + z8 + z9]
  Gamma[n1 + n6 + 2 z10 + 2 z11 + 2 z12 + 2 z13 + 2 z14 + 2 z15 + z17 + z19 + z2 + z20 +
    z22 + z23 + z25 + z26 + z27 + z28 + z29 + z3 + z4 + z5 + z6 + z7 + 2 z8 + 2 z9]
  Gamma[-10 + 5 eps + 2 n1 + n2 + n3 + 2 n4 + 2 n5 + 2 n6 + 2 n7 + n8 + n9 + 2 z1 + 2 z10 +
    2 z11 + 2 z12 + 2 z13 + 2 z14 + z15 + 2 z16 + 2 z17 + z18 + z19 + 3 z2 + 2 z20 + z21 +
    2 z22 + z23 + z24 + z25 + z26 + z27 + 2 z3 - z30 + 2 z4 + 2 z5 + 2 z6 + z7 + 3 z8 + 3 z9]) /
(Gamma[n1] Gamma[n2] Gamma[n3] Gamma[n4] Gamma[n5] Gamma[n6] Gamma[n7]
  Gamma[8 - 4 eps - n1 - n2 - n3 - n4 - n5 - n6 - n7 - n8 - n9] Gamma[n9]
  Gamma[-6 + 3 eps + n1 + n2 + n3 + n4 + n5 + n6 + n7 + n8 + n9]
  Gamma[12 - 6 eps - 2 n1 - n2 - n3 - 2 n4 - 2 n5 - 2 n6 - 2 n7 - n8 -
    n9 - z1 - z10 - z11 - z12 - z13 - z14 - z16 - z17 - 2 z2 - z20 -
    z21 - z22 - z24 + z28 + z29 - z3 + z30 - z4 - z5 - z6 - 2 z8 - 2 z9]
  Gamma[12 - 6 eps - 2 n1 - 2 n2 - n3 - 2 n4 - 2 n5 - 2 n6 - 2 n7 - n8 - n9 - 2 z1 -
    2 z10 - 2 z11 - 2 z12 - 2 z13 - 2 z14 - 2 z15 - z16 - z17 - z18 - z19 -
    2 z2 - z20 - z21 - z22 - z23 - 2 z3 - 2 z4 - 2 z5 - 2 z6 - 2 z7 - 2 z8 - 2 z9]
  Gamma[2 - eps - n2 + z12 + z13 + z14 + z16 + z17 + z2 + z20 + z22 + z24 + z25 + z26 +
    z27 + z29 - z30 + z4 + z8 + z9] Gamma[n1 + n6 + 2 z10 + 2 z11 + 2 z12 + 2 z13 +
    2 z14 + 2 z15 + z19 + z2 + z20 + z22 + z23 + z3 + z4 + z5 + z6 + z7 + 2 z8 + 2 z9])
```

```
fin = BarnesLemma[#, 1] & /@ res
```

```
>> Barnes 1st Lemma will be checked for:
 {z31, z30, z29, z28, z27, z26, z25, z24, z23, z22, z21, z20, z19, z18, z17,
  z16, z15, z14, z13, z12, z11, z10, z9, z8, z7, z6, z5, z4, z3, z2, z1} <<
   Starting with dim=31 representation...


1. Checking z31...Barnes Lemma was applied.
2. Checking z30
3. Checking z29
4. Checking z28
5. Checking z27
6. Checking z26
7. Checking z25
8. Checking z24
9. Checking z23
10. Checking z22
11. Checking z21
12. Checking z20
13. Checking z19
14. Checking z18
15. Checking z17
16. Checking z16
17. Checking z15
18. Checking z14
19. Checking z13
20. Checking z12
21. Checking z11
22. Checking z10
23. Checking z9
24. Checking z8
25. Checking z7
26. Checking z6
27. Checking z5
28. Checking z4
29. Checking z3
30. Checking z2
31. Checking z1
>> Representation after 1st Barnes Lemma:  <<

   1st Barnes Lemma was applied for: {z31}
   Obtained representation has: dim=30

{{(-1)^(n1+n2+n3+n4+n5+n6+n7+n8+n9) (-s)^(3/2 (4-2 eps)-n1-n2-n3-n4-n5-n6-n7-n8-n9) Gamma[-z1] Gamma[-z10]
     Gamma[-z11] Gamma[-z12] Gamma[-z13] Gamma[-z14] Gamma[-z15] Gamma[-z16]
     Gamma[-z17] Gamma[-z18] Gamma[-z19] Gamma[-z2] Gamma[-z20] Gamma[-z21]
     Gamma[-z22] Gamma[-z23] Gamma[-z24] Gamma[-z25] Gamma[-z26] Gamma[-z28]
     Gamma[-z29] Gamma[-z3] Gamma[-z30] Gamma[-z4] Gamma[-z5] Gamma[-z6]
     Gamma[-z7] Gamma[8 - 4 eps - n1 - n2 - n3 - n4 - 2 n5 - n6 - n7 - n8 - n9 - z1 -
       z10 - z13 - z16 - z2 - z20 - z21 - z22 - z24 + z28 + z29 - z3 + z30 - z4 - z5 - z8]
     Gamma[-z8] Gamma[-8 + 4 eps + n1 + n2 + n3 + n4 + 2 n5 + n6 + n7 + n8 + n9 + z1 + z10 +
       z13 + z16 + z2 + z20 + z21 + z22 + z24 - z27 - z28 - z29 + z3 - z30 + z4 + z5 + z8]
     Gamma[12 - 6 eps - 3 n1 - n2 - n3 - 2 n4 - 2 n5 - 3 n6 - 2 n7 - n8 - n9 - z1 - 3 z10 -
       3 z11 - 3 z12 - 3 z13 - 3 z14 - 2 z15 - z16 - 2 z17 - z19 - 3 z2 - 2 z20 - z21 - 2 z22 -
       z23 - z24 - z25 - z26 - z27 - 2 z3 + z30 - 2 z4 - 2 z5 - 2 z6 - z7 - 4 z8 - 4 z9]
     Gamma[10 - 5 eps - 2 n1 - n2 - n3 - n4 - 2 n5 - 2 n6 - 2 n7 - n8 - n9 - z1 - 2 z10 -
```

```
    2 z11 - z12 - 2 z13 - 2 z14 - z15 - z16 - z17 - z19 - 2 z2 - z20 -
      z21 - z22 - z23 - z24 - z26 - 2 z3 + z30 - 2 z4 - z5 - z6 - 2 z8 - 2 z9]
   Gamma[12 - 6 eps - 2 n1 - 2 n2 - n3 - 2 n4 - 2 n5 - 2 n6 - 2 n7 - n8 - n9 - 2 z1 -
      2 z10 - 2 z11 - z12 - z13 - z14 - z15 - z16 - z17 - z18 - z19 - 2 z2 -
      z20 - z21 - z22 - z23 + z29 - 2 z3 - z4 - 2 z5 - 2 z6 - z7 - 2 z8 - 2 z9]
   Gamma[2 - eps - n1 - n4 - n6 - z10 - z11 - 2 z12 - z13 - z14 - z15 - z17 -
      z2 - z20 - z22 - z25 - z27 - z29 - z5 - z6 - z7 - 2 z8 - 2 z9]
   Gamma[6 - 3 eps - n1 - n3 - n4 - n5 - n6 - n7 - n8 - n9 - z12 - z13 - z14 - z16 -
      z17 - z2 - z20 - z22 - z24 - z25 - z26 + z28 + z30 - z4 - z8 - z9]
   Gamma[6 - 3 eps - n1 - n2 - n3 - n4 - n5 - n6 - n7 - n8 - z1 - z10 - z11 - z12 - z13 -
      z14 - z15 - z16 - z17 - z18 - z19 - z2 - z20 - z3 - z4 - z5 - z6 - z7 - z8 - z9]
   Gamma[6 - 3 eps - n1 - n2 - n4 - n5 - n6 - n7 - n8 - n9 - z1 - z10 - z11 - z12 -
      z13 - z14 - z15 - z2 - z21 - z22 - z23 - z3 - z4 - z5 - z6 - z7 - z8 - z9]
   Gamma[-z9] Gamma[2 - eps - n2 + z12 + z13 + z14 + z16 + z17 + z2 +
      z20 + z22 + z24 + z25 + z26 + z27 + z29 + z4 + z8 + z9]
   Gamma[n1 + z10 + z11 + z12 + z13 + z14 + z15 + z19 + z2 + z22 + z3 + z4 + z8 + z9]
   Gamma[n6 + z10 + z11 + z12 + z13 + z14 + z15 + z20 + z23 + z5 + z6 + z7 + z8 + z9]
   Gamma[-6 + 3 eps + n1 + n2 + n3 + n4 + n5 + n6 + n7 + n8 + n9 + z1 +
      z10 + z11 + z12 + z13 + z14 + z15 + z16 + z17 + z18 + z19 + z2 + z20 +
      z21 + z22 + z23 + z24 + z25 + z26 + z3 + z4 + z5 + z6 + z7 + z8 + z9]
   Gamma[n1 + n6 + 2 z10 + 2 z11 + 2 z12 + 2 z13 + 2 z14 + 2 z15 + z17 + z19 + z2 + z20 +
      z22 + z23 + z25 + z26 + z27 + z28 + z29 + z3 + z4 + z5 + z6 + z7 + 2 z8 + 2 z9]
   Gamma[-10 + 5 eps + 2 n1 + n2 + n3 + 2 n4 + 2 n5 + 2 n6 + 2 n7 + n8 + n9 + 2 z1 + 2 z10 +
      2 z11 + 2 z12 + 2 z13 + 2 z14 + z15 + 2 z16 + 2 z17 + z18 + z19 + 3 z2 + 2 z20 + z21 +
      2 z22 + z23 + z24 + z25 + z26 + z27 + 2 z3 - z30 + 2 z4 + 2 z5 + 2 z6 + z7 + 3 z8 + 3 z9]) /
  (Gamma[n1] Gamma[n2] Gamma[n3] Gamma[n4] Gamma[n5] Gamma[n6] Gamma[n7]
   Gamma[8 - 4 eps - n1 - n2 - n3 - n4 - n5 - n6 - n7 - n8 - n9] Gamma[n9]
   Gamma[12 - 6 eps - 3 n1 - n2 - n3 - 2 n4 - 2 n5 - 3 n6 - 2 n7 - n8 - n9 - z1 - 3 z10 -
      3 z11 - 3 z12 - 3 z13 - 3 z14 - 2 z15 - z16 - 2 z17 - z19 - 3 z2 - 2 z20 - z21 - 2 z22 -
      z23 - z24 - z25 - z26 - z27 - z29 - 2 z3 + z30 - 2 z4 - 2 z5 - 2 z6 - z7 - 4 z8 - 4 z9]
   Gamma[12 - 6 eps - 2 n1 - n2 - n3 - 2 n4 - 2 n5 - 2 n6 - 2 n7 - n8 - n9 - z1 -
      z10 - z11 - z12 - z13 - z14 - z16 - z17 - 2 z2 - z20 - z21 -
      z22 - z24 + z28 + z29 - z3 + z30 - z4 - z5 - z6 - 2 z8 - 2 z9]
   Gamma[12 - 6 eps - 2 n1 - 2 n2 - n3 - 2 n4 - 2 n5 - 2 n6 - 2 n7 - n8 - n9 - 2 z1 -
      2 z10 - 2 z11 - 2 z12 - 2 z13 - 2 z14 - 2 z15 - z16 - z17 - z18 - z19 -
      2 z2 - z20 - z21 - z22 - z23 - 2 z3 - 2 z4 - 2 z5 - 2 z6 - 2 z7 - 2 z8 - 2 z9]
   Gamma[2 - eps - n2 + z12 + z13 + z14 + z16 + z17 + z2 + z20 + z22 + z24 + z25 + z26 +
      z27 + z29 - z30 + z4 + z8 + z9] Gamma[n1 + n6 + 2 z10 + 2 z11 + 2 z12 + 2 z13 +
      2 z14 + 2 z15 + z19 + z2 + z20 + z22 + z23 + z3 + z4 + z5 + z6 + z7 + 2 z8 + 2 z9])}

finres = fin /. {n1 → 1, n2 → 1, n3 → 1, n4 → 1, n5 → 1, n6 → 1, n7 → 1, n8 → 1, n9 → 1}
```

```
{-((-s)^(-9+3/2 (4-2 eps)) Gamma[-z1] Gamma[-z10] Gamma[-z11] Gamma[-z12] Gamma[-z13]
      Gamma[-z14] Gamma[-z15] Gamma[-z16] Gamma[-z17] Gamma[-z18] Gamma[-z19]
      Gamma[-z2] Gamma[-z20] Gamma[-z21] Gamma[-z22] Gamma[-z23] Gamma[-z24]
      Gamma[-z25] Gamma[-z26] Gamma[-z28] Gamma[-z29] Gamma[-z3] Gamma[-z30]
      Gamma[-z4] Gamma[-z5] Gamma[-z6] Gamma[-z7] Gamma[-2-4 eps-z1-z10-
        z13-z16-z2-z20-z21-z22-z24+z28+z29-z3+z30-z4-z5-z8]
      Gamma[-z8] Gamma[2+4 eps+z1+z10+z13+z16+z2+z20+z21+z22+z24-
        z27-z28-z29+z3-z30+z4+z5+z8] Gamma[-4-6 eps-z1-3 z10-3 z11-
        3 z12-3 z13-3 z14-2 z15-z16-2 z17-z19-3 z2-2 z20-z21-2 z22-
        z23-z24-z25-z26-z27-2 z3+z30-2 z4-2 z5-2 z6-z7-4 z8-4 z9]
      Gamma[-3-5 eps-z1-2 z10-2 z11-z12-2 z13-2 z14-z15-z16-z17-z19-
        2 z2-z20-z21-z22-z23-z24-z26-2 z3+z30-2 z4-z5-z6-2 z8-2 z9]
      Gamma[-3-6 eps-2 z1-2 z10-2 z11-z12-z13-z14-z15-z16-z17-z18-
        z19-2 z2-z20-z21-z22-z23+z29-2 z3-z4-2 z5-2 z6-z7-2 z8-2 z9]
      Gamma[-1-eps-z10-z11-2 z12-z13-z14-z15-z17-z2-z20-z22-
        z25-z27-z29-z5-z6-z7-2 z8-2 z9] Gamma[-2-3 eps-z12-z13-
        z14-z16-z17-z2-z20-z22-z24-z25-z26+z28+z30-z4-z8-z9]
      Gamma[-2-3 eps-z1-z10-z11-z12-z13-z14-z15-z16-z17-
        z18-z19-z2-z20-z3-z4-z5-z6-z7-z8-z9]
      Gamma[-2-3 eps-z1-z10-z11-z12-z13-z14-z15-z2-z21-z22-z23-
        z3-z4-z5-z6-z7-z8-z9] Gamma[-z9] Gamma[1-eps+z12+z13+
        z14+z16+z17+z2+z20+z22+z24+z25+z26+z27+z29+z4+z8+z9]
      Gamma[1+z10+z11+z12+z13+z14+z15+z19+z2+z22+z3+z4+z8+z9]
      Gamma[1+z10+z11+z12+z13+z14+z15+z20+z23+z5+z6+z7+z8+z9]
      Gamma[3+3 eps+z1+z10+z11+z12+z13+z14+z15+z16+z17+z18+z19+
        z2+z20+z21+z22+z23+z24+z25+z26+z3+z4+z5+z6+z7+z8+z9]
      Gamma[2+2 z10+2 z11+2 z12+2 z13+2 z14+2 z15+z17+z19+z2+z20+
        z22+z23+z25+z26+z27+z28+z29+z3+z4+z5+z6+z7+2 z8+2 z9]
      Gamma[4+5 eps+2 z1+2 z10+2 z11+2 z12+2 z13+2 z14+z15+2 z16+
        2 z17+z18+z19+3 z2+2 z20+z21+2 z22+z23+z24+z25+
        z26+z27+2 z3-z30+2 z4+2 z5+2 z6+z7+3 z8+3 z9]) /
    (Gamma[-1-4 eps] Gamma[-4-6 eps-z1-3 z10-3 z11-3 z12-3 z13-3 z14-
        2 z15-z16-2 z17-z19-3 z2-2 z20-z21-2 z22-z23-z24-z25-
        z26-z27-z29-2 z3+z30-2 z4-2 z5-2 z6-z7-4 z8-4 z9]
      Gamma[-2-6 eps-z1-z10-z11-z12-z13-z14-z16-z17-2 z2-z20-
        z21-z22-z24+z28+z29-z3+z30-z4-z5-z6-2 z8-2 z9]
      Gamma[-3-6 eps-2 z1-2 z10-2 z11-2 z12-2 z13-2 z14-2 z15-z16-z17-
        z18-z19-2 z2-z20-z21-z22-z23-2 z3-2 z4-2 z5-2 z6-2 z7-2 z8-2 z9]
      Gamma[1-eps+z12+z13+z14+z16+z17+z2+z20+z22+z24+z25+z26+
        z27+z29-z30+z4+z8+z9] Gamma[2+2 z10+2 z11+2 z12+2 z13+2 z14+
        2 z15+z19+z2+z20+z22+z23+z3+z4+z5+z6+z7+2 z8+2 z9])}

Quit[]
```